

The background features a large, faint watermark of the Brown University crest. The crest includes a sun with a face at the top, a shield with a red cross, and a banner at the bottom with the Latin motto "IN DEO SPERAMUS".

Informed Search

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Recall: Search



5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9



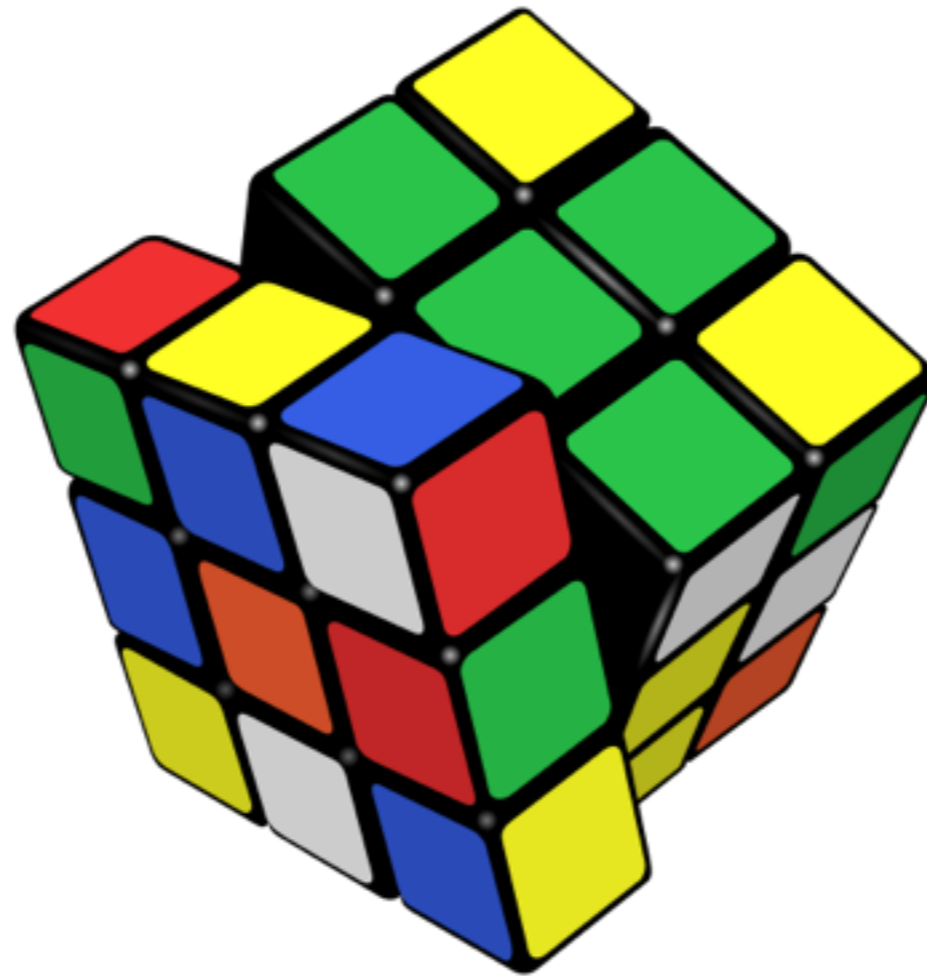
5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

Basic to problem solving:

- *How to take action to reach a goal?*

Informed Search

What if we *know something* about the search?



How should we include that knowledge?
In what form should it be expressed to be useful?



Formal Definition

Set of states S

Start state $s \in S$

Set of actions A and action rules $a(s) \rightarrow s'$

Goal test $g(s) \rightarrow \{0, 1\}$

Cost function $C(s, a, s') \rightarrow \mathbb{R}^+$

So a search problem is specified by a tuple, (S, s, A, g, C) .



Problem Statement

Find a sequence of actions a_1, \dots, a_n
and corresponding states s_1, \dots, s_n

... such that:

$$s_0 = s$$

$$s_i = a_i(s_{i-1}), i = 1, \dots, n$$

$$g(s_n) = 1$$

start state

legal moves

end at the goal

while minimizing:

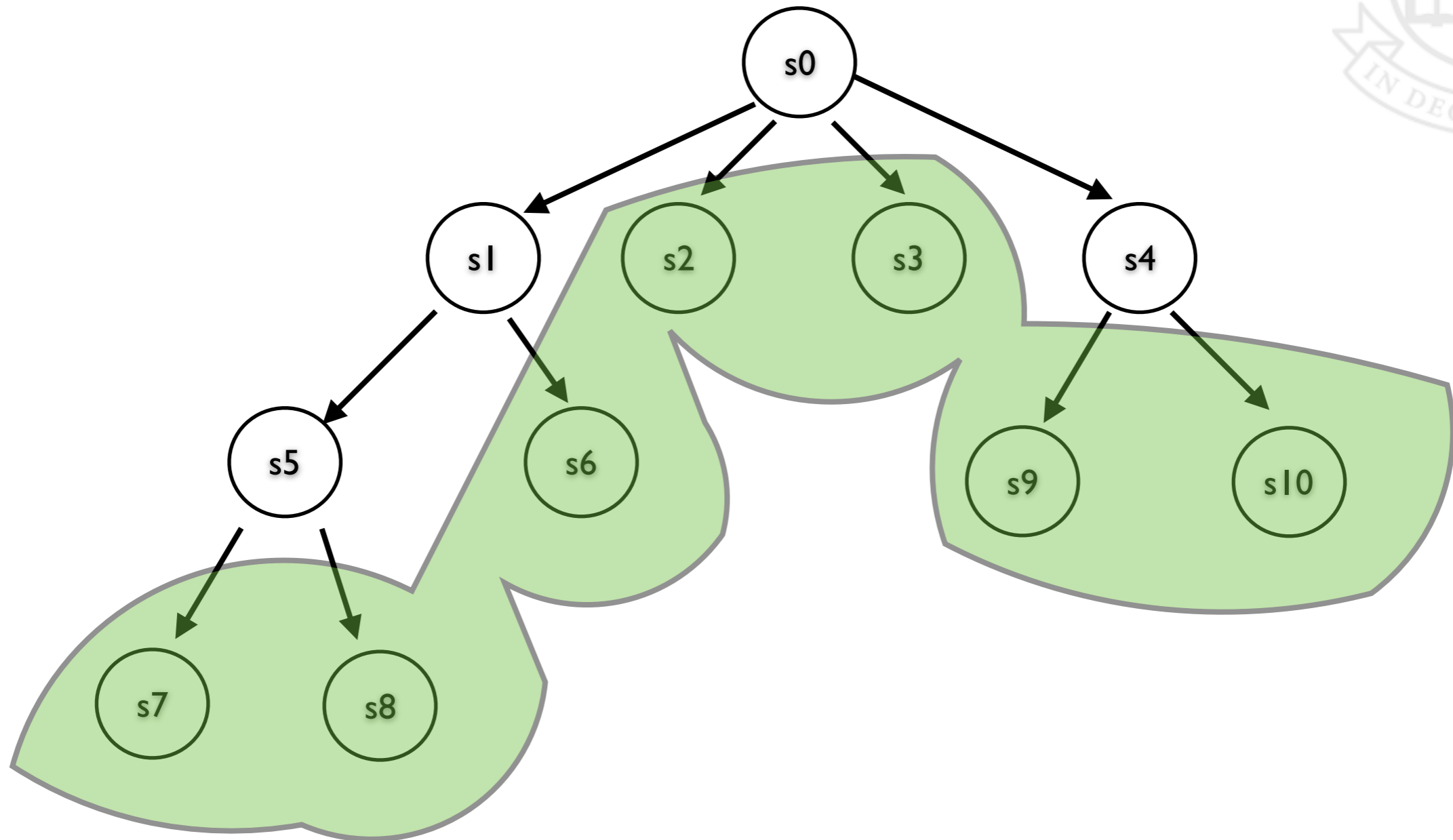
$$\sum_{i=1}^n C(s_{i-1}, a, s_i)$$

minimize sum of costs - *rational agent*



The Frontier

Key thing in search is *managing the frontier*.



Uninformed Searches

Simple strategy for choosing next node:

- Choose the shallowest one (**breadth-first**)
- Choose the deepest one (**depth-first**)

Neither guaranteed to find the least-cost path.

What if we chose the one with *lowest cost*?



Uniform-Cost

Order the nodes in the frontier by *cost-so-far*

- Cost from the start state to that node.

Open the next node with the smallest cost-so-far

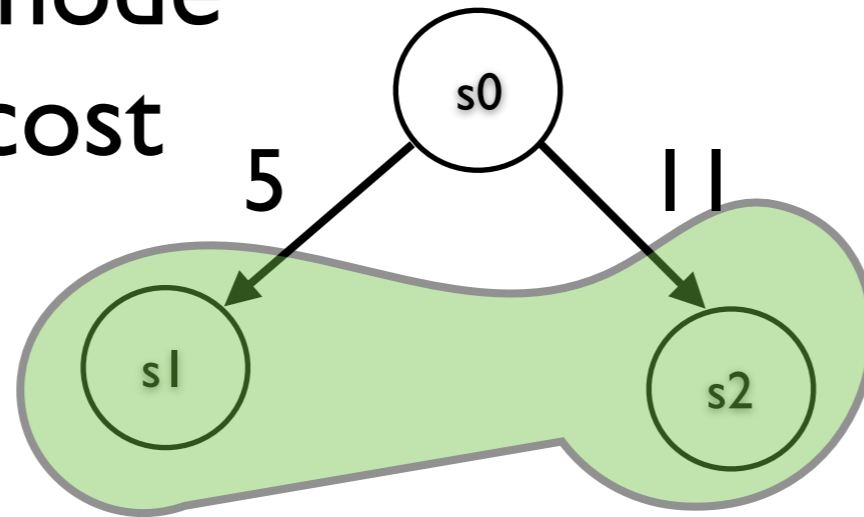
- Optimal solution
- Complete (provided no negative costs)



Uniform-Cost

Expand cheapest node

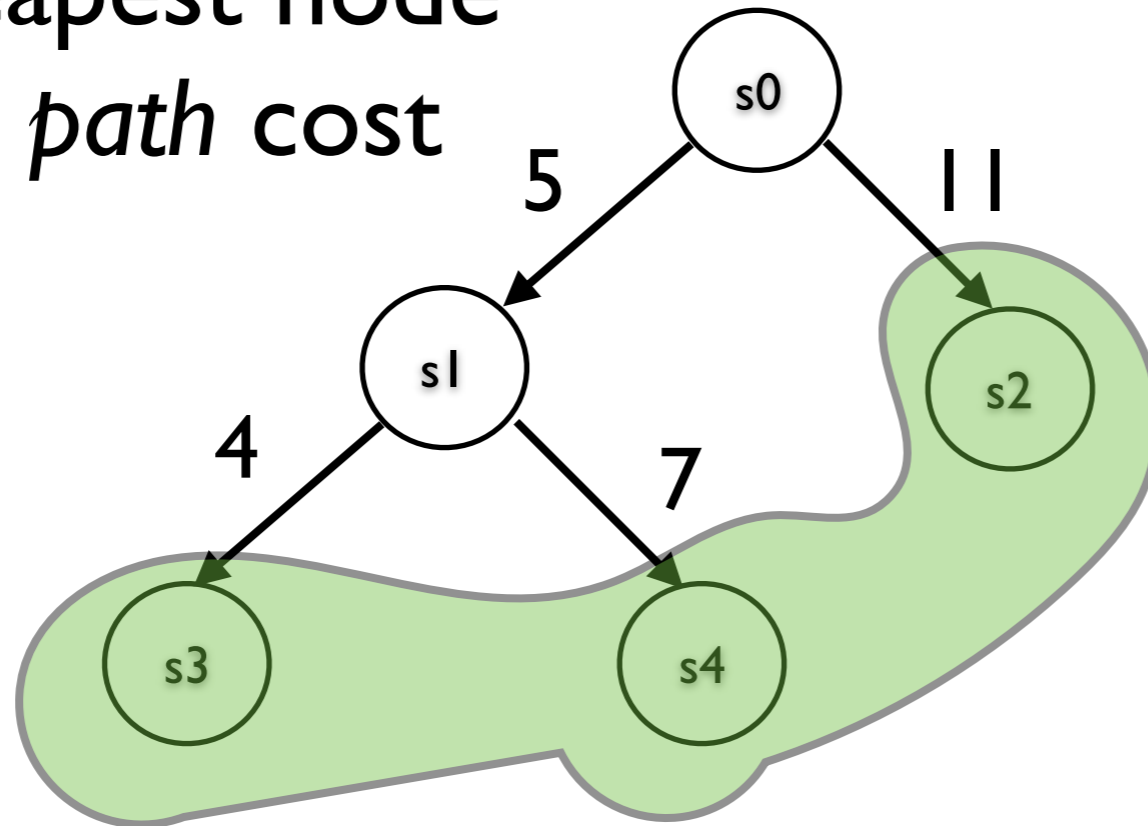
Use *whole path* cost



Uniform-Cost

Expand cheapest node

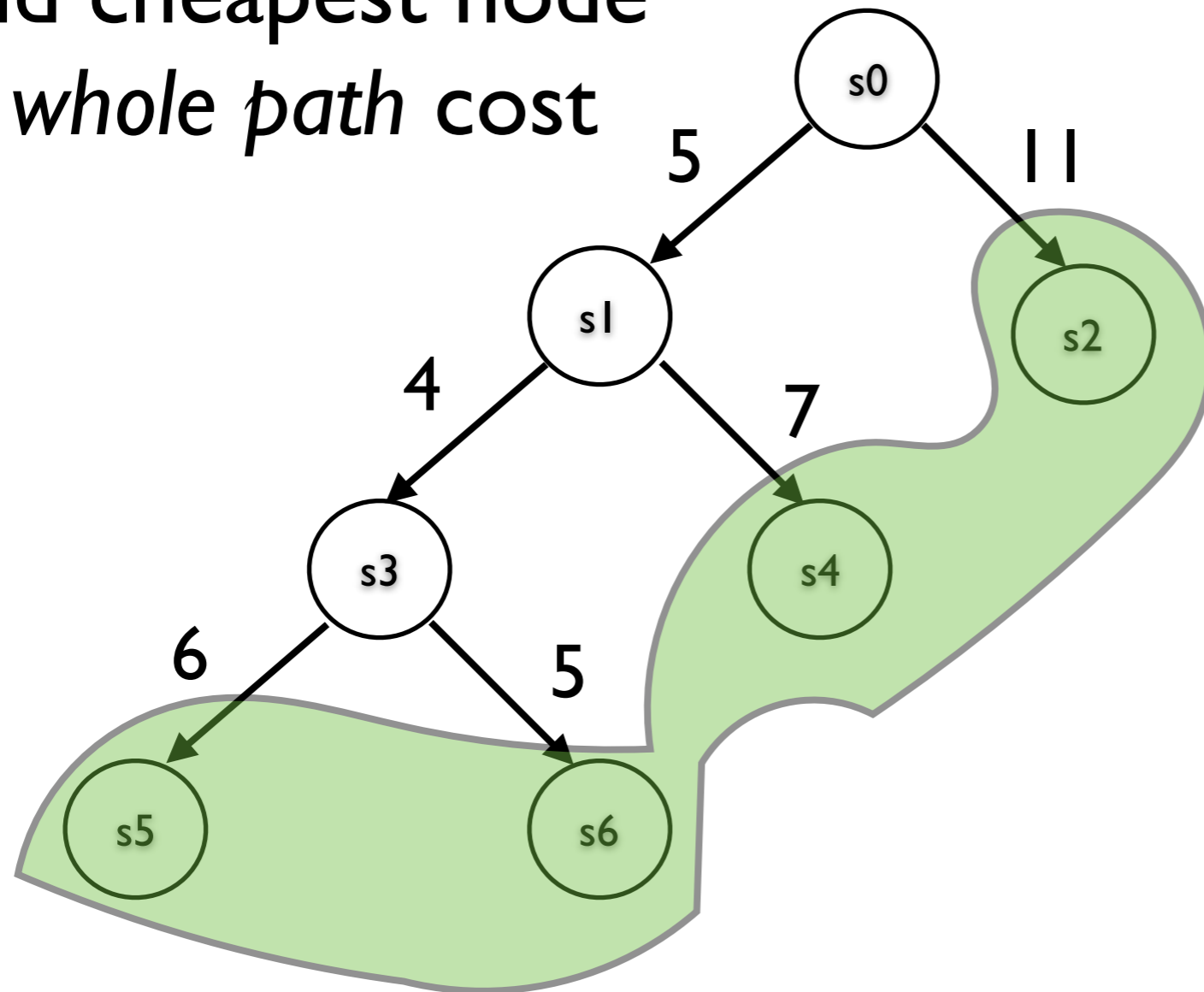
Use *whole path cost*



Uniform-Cost

Expand cheapest node

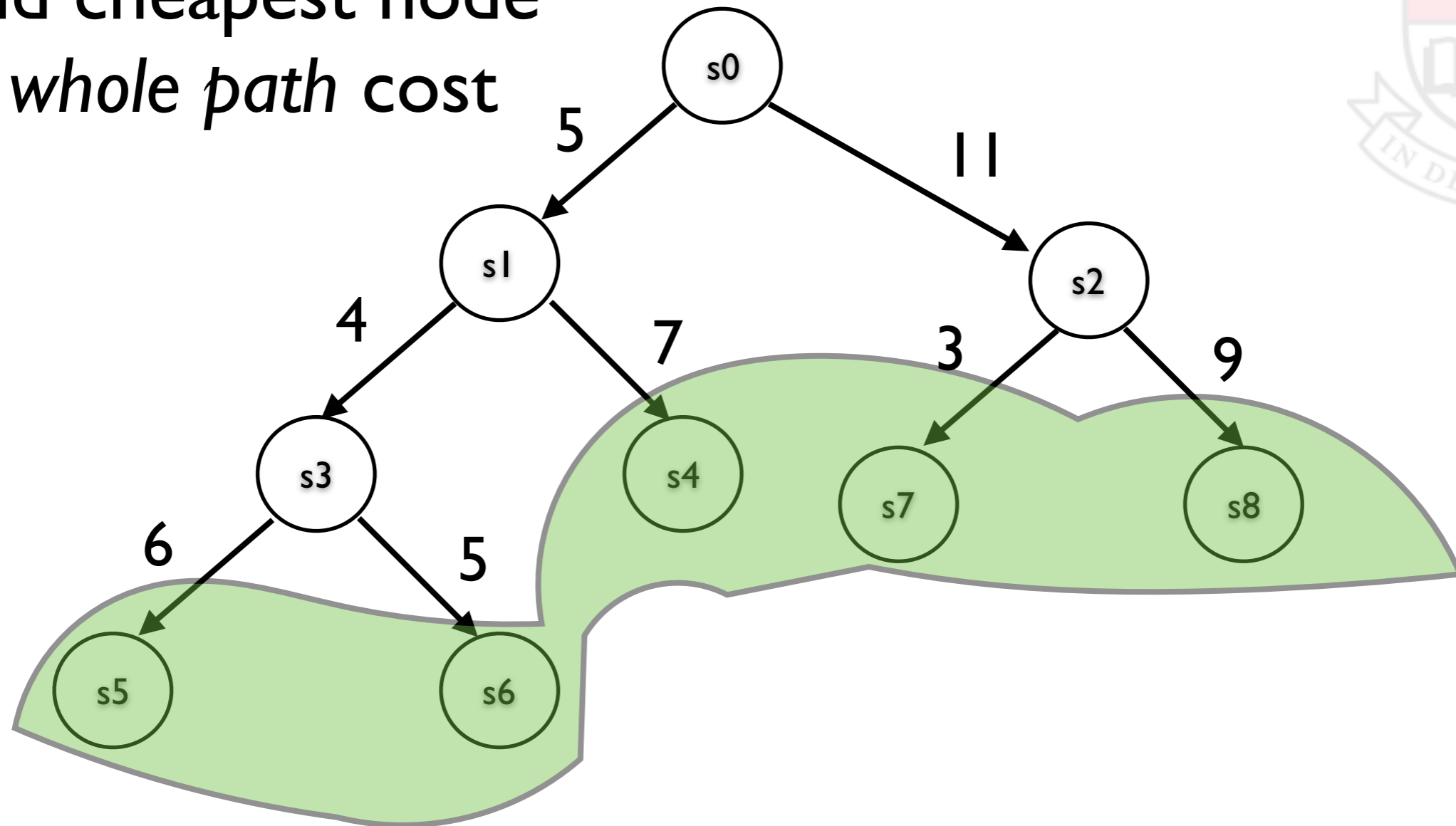
Use *whole path cost*



Uniform-Cost

Expand cheapest node

Use *whole path cost*



What's the Insight?

The *cost-so-far* tells us how much it cost to get to a node.

- Go to cheapest nodes first.

How might we prove that this is *optimal*?



What's the Insight?

The *cost-so-far* tells us how much it cost to get to a node.

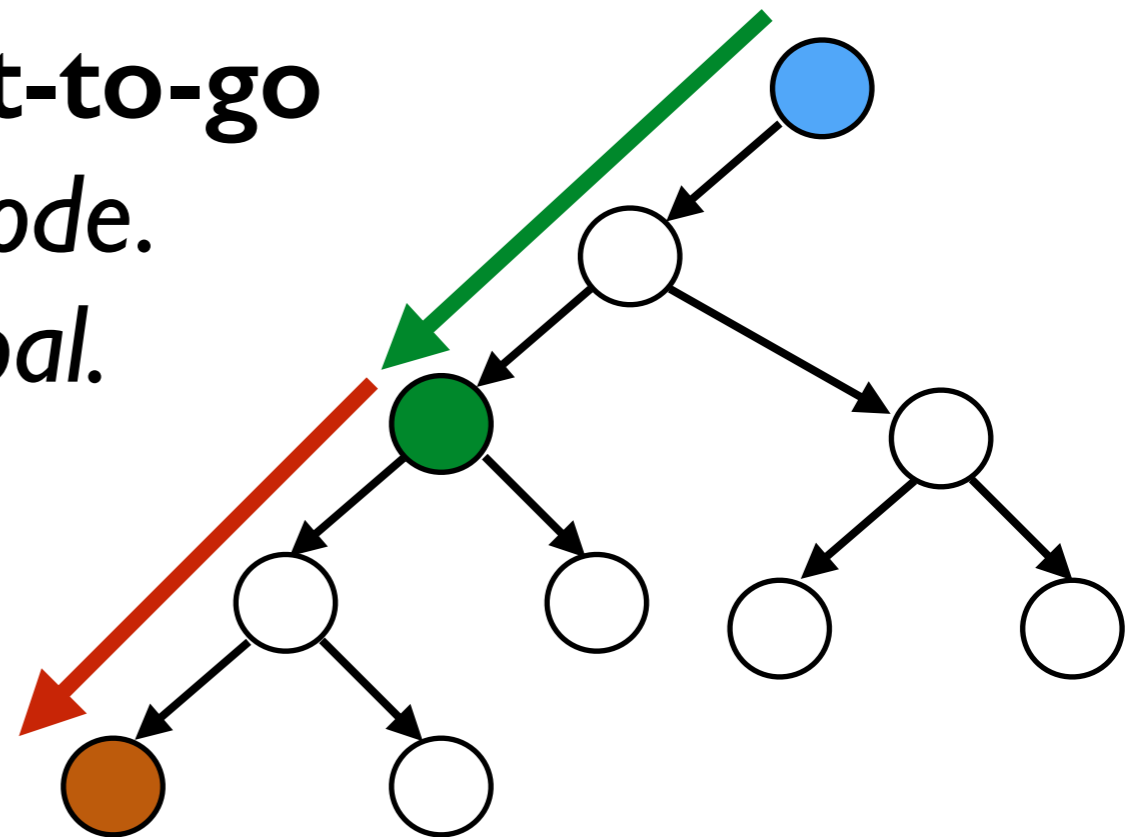
- Go to cheapest nodes first.

What remains?

Total cost = **cost-so-far** + **cost-to-go**

Cost-so-far: cost from start to node.

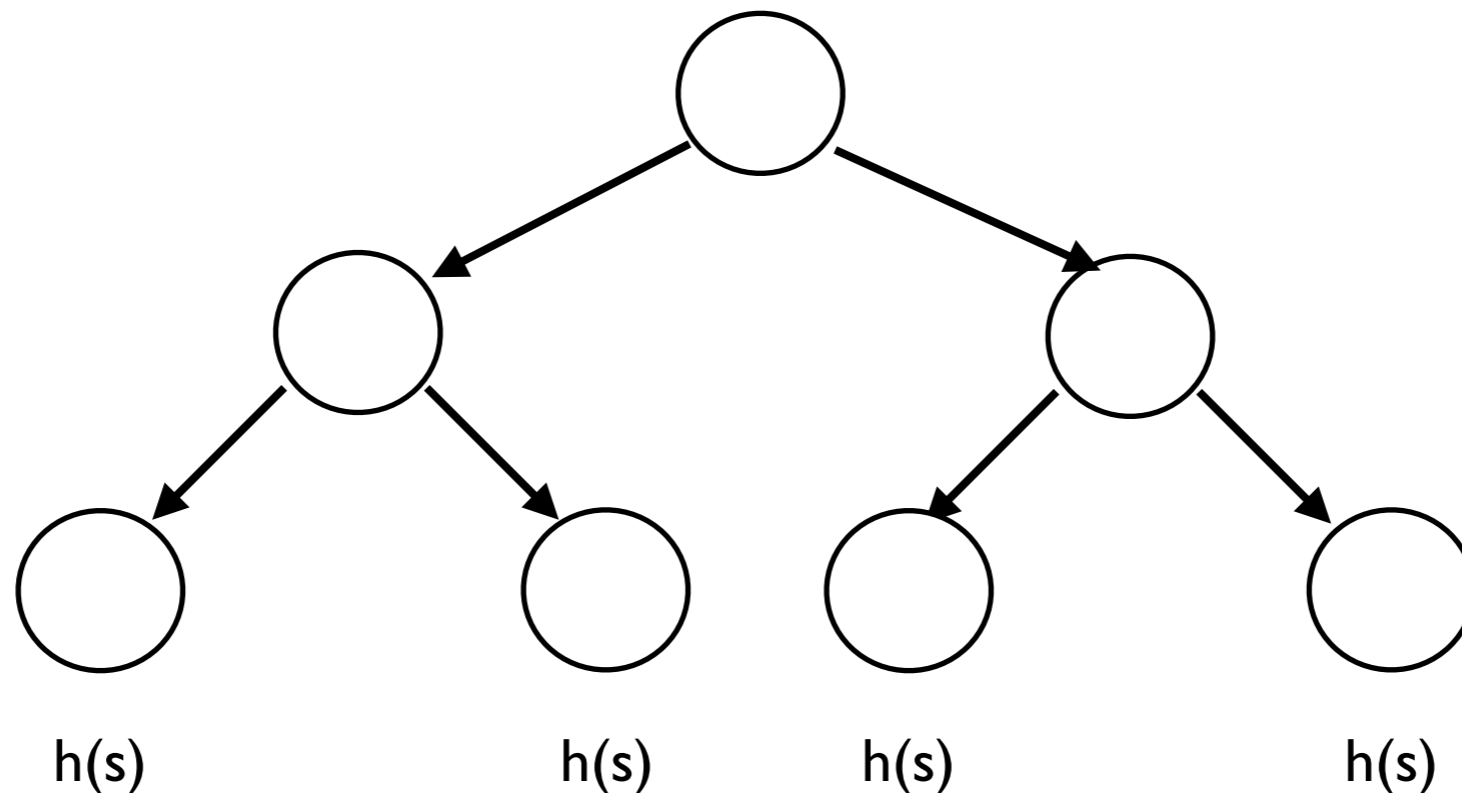
Cost-to-go: cost from node to goal.



Informed Search

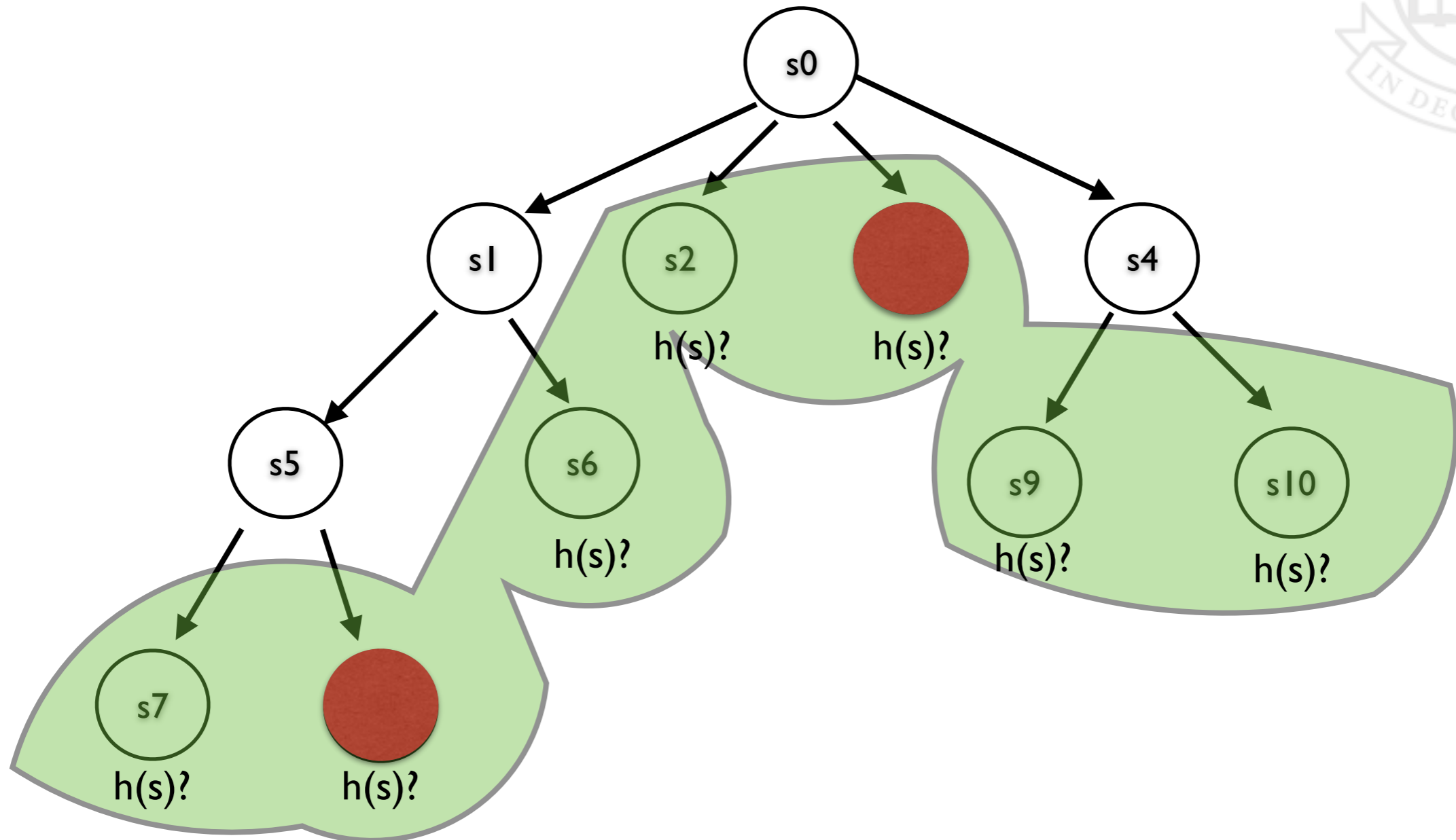
Key idea: *heuristic function*.

- $h(s)$ - estimates cost-to-go
- Cost to go *from* state *to* solution.
- Problem specific (hence *informed*)



Greed

What if we expand the node with lowest $h(s)$?



Informed Search: A*

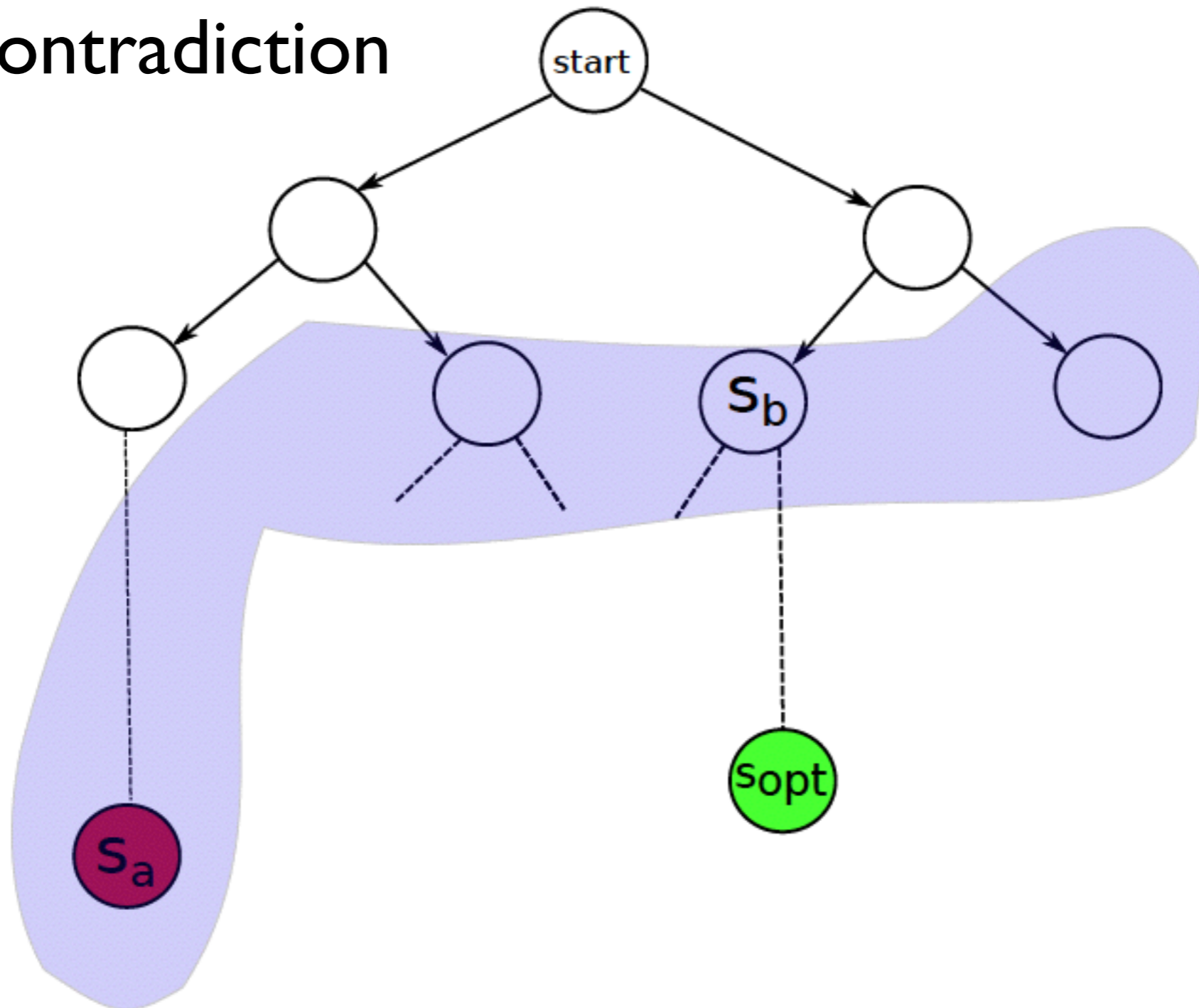
A* algorithm:

- $g(s)$ - cost so far (start to s).
- Expand s that minimizes $g(s) + h(s)$ **both**
- Manage frontier as priority queue.
- Admissible heuristic: *never overestimates cost.*
$$h(s) \leq h^*(s)$$
- $h(s) = 0$ if s is a goal state, so $g(s) + h(s) = c(s)$
- **If h is admissible, A* is optimal.**
- If $h(s)$ is exact, runs in $O(bd)$ time.



Admissible Heuristics

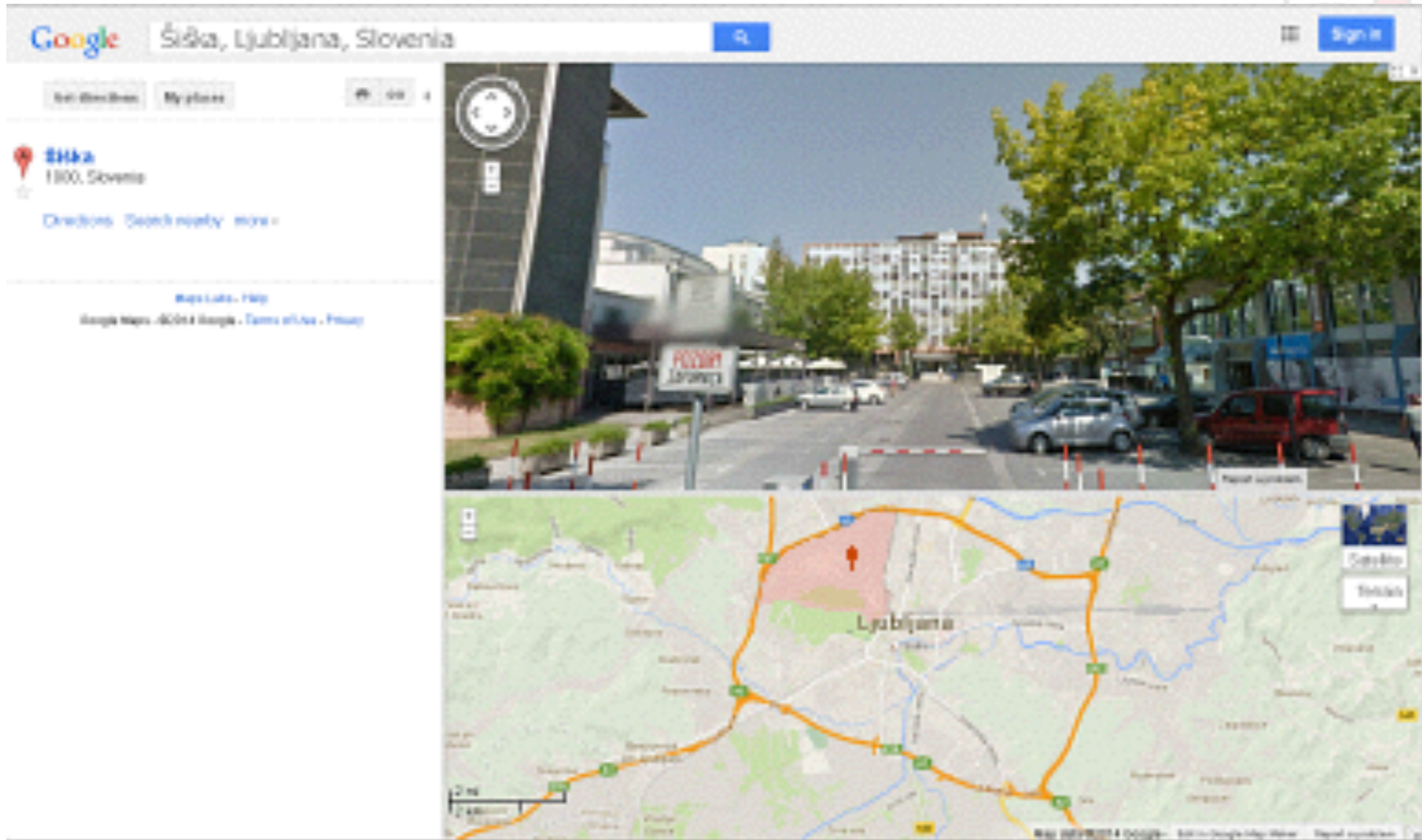
Optimal solution
Proof by contradiction



Example Heuristic



Example Heuristics



More on Heuristics

Heuristic h_1 dominates h_2 if $h_1(s) \geq h_2(s)$ for all s .

- Is h_1 or h_2 better? (If they're both admissible.)

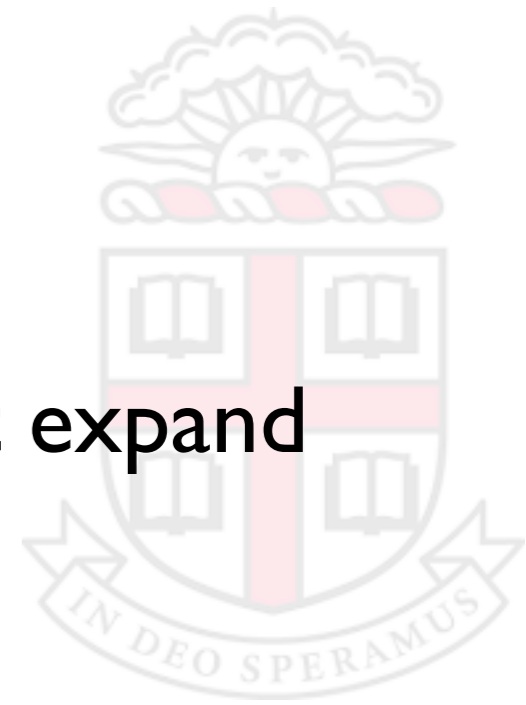
How might you combine two heuristics?



More on Heuristics

A^* is optimally efficient: any algorithm using h must expand the nodes A^* expands.

Why?



More on Heuristics

Ideal heuristics:

- Fast to compute.
- Close to real costs.
- Some programs *automatically generate* heuristics.

