Knowledge Representation and Reasoning

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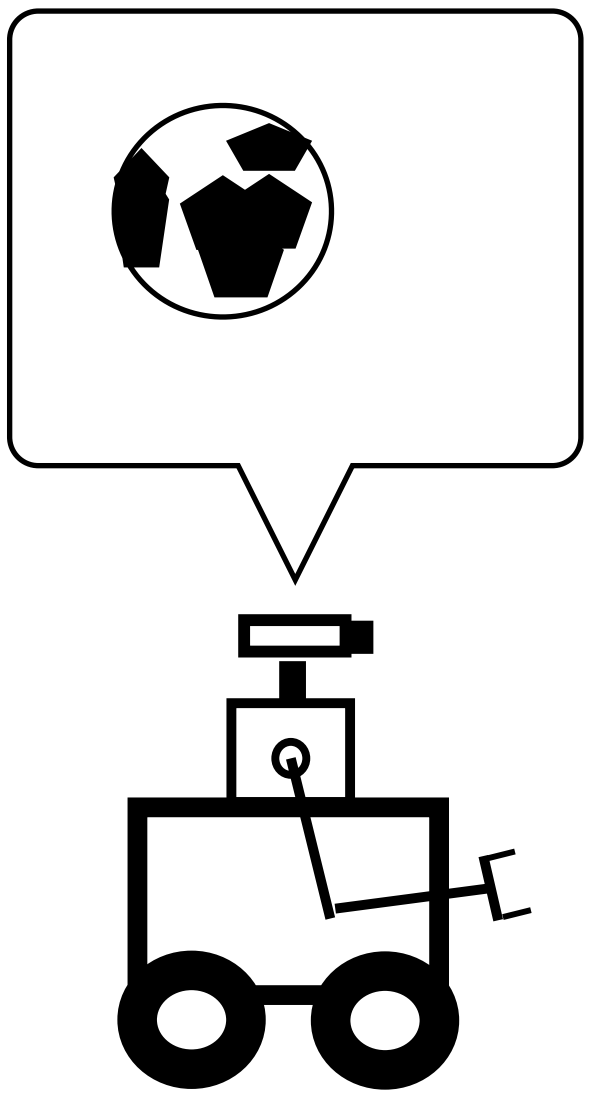
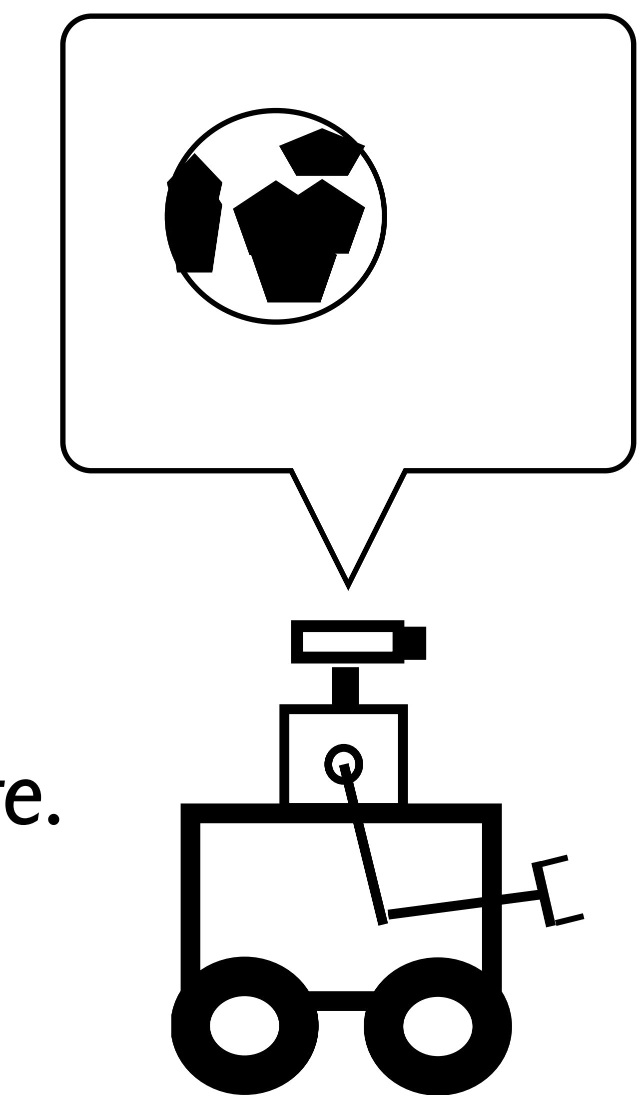
Knowledge



Representation and Reasoning

Represent knowledge about the world.

- Representation language.
- Knowledge base.
- Declarative - *facts* and *rules*.

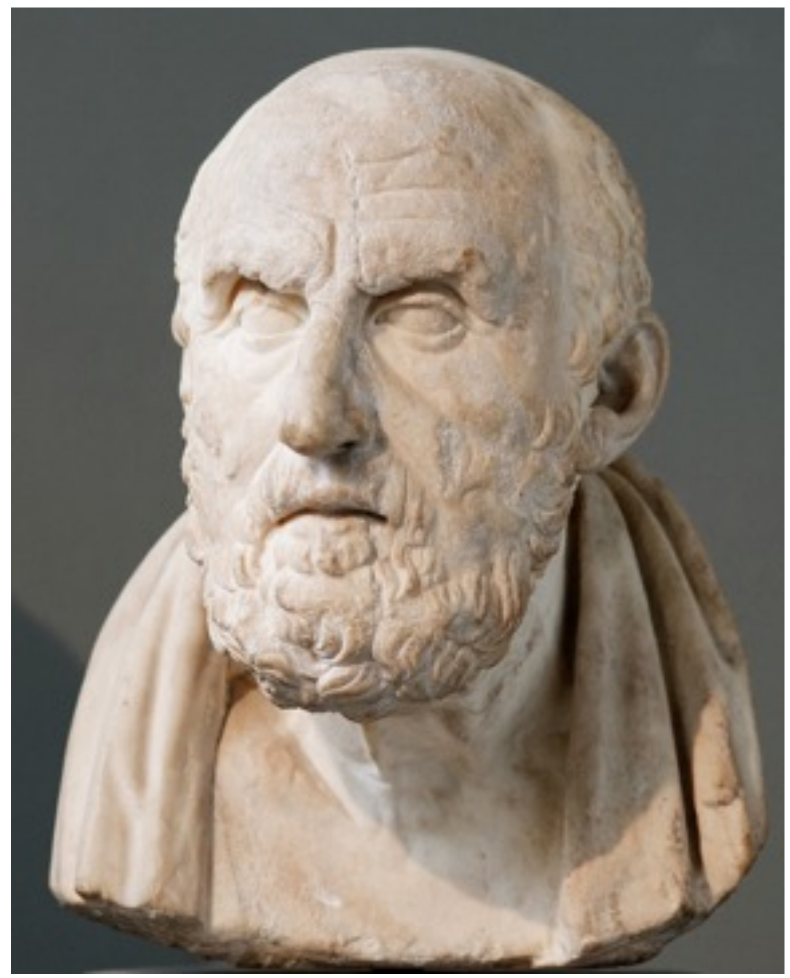


Reason using that represented knowledge.

- Often *asking questions*.
- Inference procedure.
- Heavily dependent on *representation language*.

Propositional Logic

Representation language and *set of inference rules* for reasoning about facts that are either **true** or **false**.



Chrysippus of Soli, 3rd century BC

"that which is capable of being denied or affirmed as it is in itself"

Propositional Logic

A proposition is:

- A possible *fact* about the world.
- Either **true** or **false**.
- *We may not know which.*
- Can be combined with logical connectives.

May change over time:

- It's Raining
- It's Cloudy
- The Patriots are the Super Bowl champions.

Propositional Logic

Can combine propositions using **logical operators** to make **sentences** ***(syntax vs. semantics)*:**

$\neg A$ (not A - **A is *False***)

$A \vee B$ (A or B - **one (or both) of A or B is *True***)

$A \wedge B$ (A and B - **both A and B are *True***)

$A \implies B$ (A implies B - **if A is *True*, so is B**)

$A \iff B$ (A iff B - **A and B both *True* or both *False***)

Two uses of sentence:

- Fact
- Question

Knowledge Base

A list of *sentences that apply to the world*.

For example:

$$Cold$$

$$\neg Raining$$

$$(Raining \vee Cloudy)$$

$$Cold \iff \neg Hot$$

A knowledge base describes *a set of worlds in which these facts and rules are true*.

Knowledge Base

A *model* is a formalization of a "world":

- Set the value of every variable in the KB to *True* or *False*.
- 2^n models possible for n propositions.

Proposition	**Value**
Cold	False
Raining	False
Cloudy	False
Hot	False

Proposition	**Value**
Cold	True
Raining	False
Cloudy	False
Hot	False

...

Proposition	**Value**
Cold	True
Raining	True
Cloudy	True
Hot	True

Models and Sentences

Each sentence has a *truth value* in each model.

Proposition	Value
Cold	True
Raining	False
Cloudy	True
Hot	True

If sentence *a* is true in model *m*, then *m* **satisfies** (or **is a model of**) *a*.

$Cold$	True
$\neg Raining$	True
$(Raining \vee Cloudy)$	True
$Cold \iff \neg Hot$	False

Models and Worlds

The KB specifies a subset of all possible models - those that satisfy all sentences in the KB.

$$Cold$$
$$\neg Raining$$
$$(Raining \lor Cloudy)$$
$$Cold \iff \neg Hot$$

Proposition	Value
Cold	False
Raining	False
Cloudy	False
Hot	False

Proposition	Value
Cold	True
Raining	False
Cloudy	True
Hot	False

...

Proposition	Value
Cold	True
Raining	True
Cloudy	True
Hot	True

Each new piece of knowledge narrows down the set of possible models.

Summary

Knowledge Base

- Set of facts *asserted to be true* about the world.

Model

- Formalization of "the world".
- An assignment to values to all variables.

Satisfaction

- Satisfies a sentence if that sentence is true in the model.
- Satisfies a KB if all sentences sure in model.
- Knowledge in the KB *narrows down* the set of possible world models.

Inference

So if we have a KB, then what?

Given:

$$Cold$$

$$\neg Raining$$

$$(Raining \vee Cloudy)$$

$$Cold \iff \neg Hot$$

We'd like to ask it *questions*.

… we can ask: $Hot?$

Inference: process of deriving new facts from given facts.

Logical Inference

Take a KB, and produce new sentences of knowledge.

Inference algorithms: search process to find a proof of *Q* using a set of *inference rules*.

Desirable properties:

- Soundness (or truth-preserving)
- Complete

Inference (Formally)

KB *A* **entails** sentence *B* → $A \models B$
if and only if:
every model which satisfies *A*, satisfies *B*.

In other words: if *A* is true then *B* **must be true**.
Only conclusions you can make about the true world.

Most frequent form of inference: $KB \models Q$

That's nice, but how do we compute?

Inference (formally)

Could just enumerate worlds …

✓ ✗ Knowledge Base

✓ ✗ Query Sentence

Proposition	Value
Cold	False
Raining	False
Cloudy	False
Hot	False

Proposition	Value
Cold	True
Raining	True
Cloudy	True
Hot	True

Proposition	Value
Cold	True
Raining	True
Cloudy	True
Hot	True

Proposition	Value
Cold	True
Raining	False
Cloudy	False
Hot	False

OK

Not OK

Inference Rules

Form	Description
$(A \wedge B) \equiv (B \wedge A)$	Commutivity of $\wedge$.
$(A \vee B) \equiv (B \vee A)$	Commutivity of $\vee$.
$((A \wedge B) \wedge C) \equiv (A \wedge (B \wedge C))$	Associativity of $\wedge$.
$((A \vee B) \vee C) \equiv (A \vee (B \vee C))$	Associativity of $\vee$.
$\neg(\neg A) \equiv A$	Double negative elimination.
$(A \Rightarrow B) \equiv (\neg B \Rightarrow \neg A)$	Contraposition.
$(A \Rightarrow B) \equiv (\neg A \vee B)$	Implication elimination.
$(A \Leftrightarrow B) \equiv (A \Rightarrow B) \wedge (B \Rightarrow A)$	Biconditional elimination.
$\neg(A \wedge B) \equiv (\neg A \vee \neg B)$	De Morgan.
$\neg(A \vee B) \equiv (\neg A \wedge \neg B)$	De Morgan.
$(A \wedge (B \vee C)) \equiv ((A \wedge B) \vee (A \wedge C))$	Distributivity of $\wedge$ over $\vee$.
$(A \vee (B \wedge C)) \equiv ((A \vee B) \wedge (B \vee C))$	Distributivity of $\vee$ over $\wedge$.

If $A \implies B$, and *A* is true, then *B* is true.

If $A \wedge B$, then *A* is true, and *B* is true.

Inference Rules

Often written in form:

Start with

$$\frac{A \vee B, \neg B}{A}$$

Given this knowledge

can infer this

Proofs

For example, given KB:

$$Cold$$

$$\neg Raining$$

$$(Raining \vee Cloudy)$$

$$Cold \iff \neg Hot$$

We ask:

$$Hot?$$

Inference:

$$Cold = True$$

$$True \iff \neg Hot$$

$$\neg Hot = True$$

$$Hot = False$$

Inference …

We want to *start* somewhere (KB).
We'd like to *apply some rules*.
But there are lots of *ways we might go*.
… in order to reach some *goal* (sentence).

Does that sound familiar?

Inference as search:

Set of states	True sentences
Start state	KB
Set of actions and action rules	Inference rules
Goal test	Q in sentences?
Cost function	1 per rule

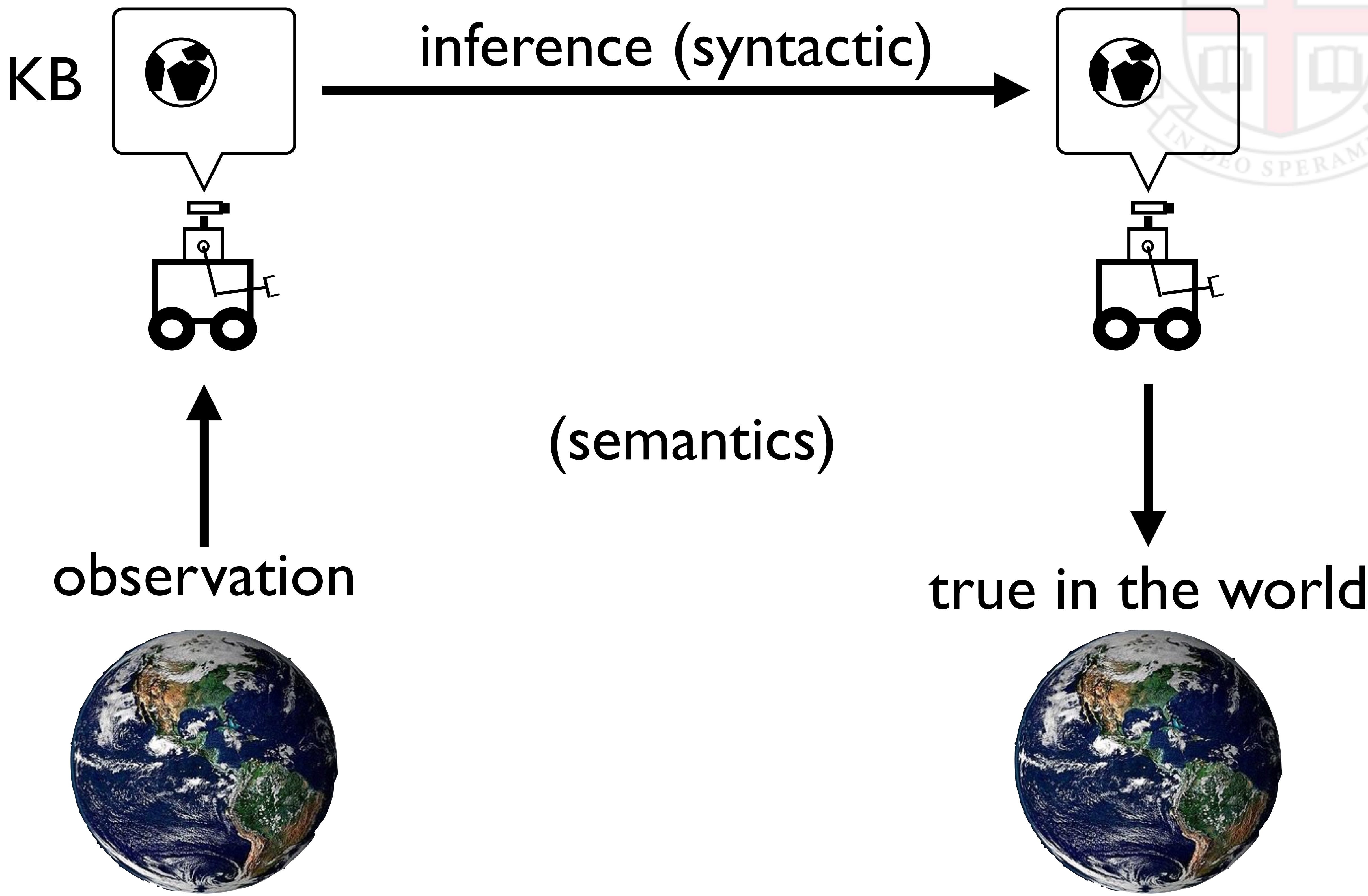
Resolution

The following inference rule is **both sound and complete:**

$$\frac{a_1 \vee \ldots \vee a_{i-1} \vee c \vee a_{i+1} \vee \ldots \vee a_n, \quad b_1 \vee \ldots \vee b_{j-1} \vee \neg c \vee b_{j+1} \vee \ldots \vee b_m}{a_1 \vee \ldots \vee a_{i-1} \vee a_{i+1} \vee \ldots \vee a_n \vee b_1 \vee \ldots \vee b_{j-1} \vee b_{j+1} \vee \ldots \vee b_m}$$

This is called **resolution**. It is sound and complete when combined with a sound and complete search algorithm.

The World and the Model



DENDRAL and MYCIN

"**Expert Systems**" - knowledge based.

DENDRAL: (Feigenbaum et al. ~1965)

- Identify unknown organic molecules
- Eliminate most "chemically implausible" hypotheses.

MYCIN: (Shortliffe et al., 1970s)

- Identify bacteria causing severe infections.
- "research indicated that it proposed an acceptable therapy in about 69% of cases, which was *better than the performance of infectious disease experts*."

Major issue: **the Knowledge Bottleneck.**