Markov Logic Networks

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(Joint work with Matt Richardson)

Overview

- Representation
- Inference
- Learning
- Applications

Markov Logic Networks

- A logical KB is a set of **hard constraints** on the set of possible worlds
- Let's make them **soft constraints**:
  When a world violates a formula,
  It becomes less probable, not impossible
- Give each formula a **weight**
  (Higher weight $\Rightarrow$ Stronger constraint)

$$P(world) \propto \exp\left(\sum \text{weights of formulas it satisfies}\right)$$

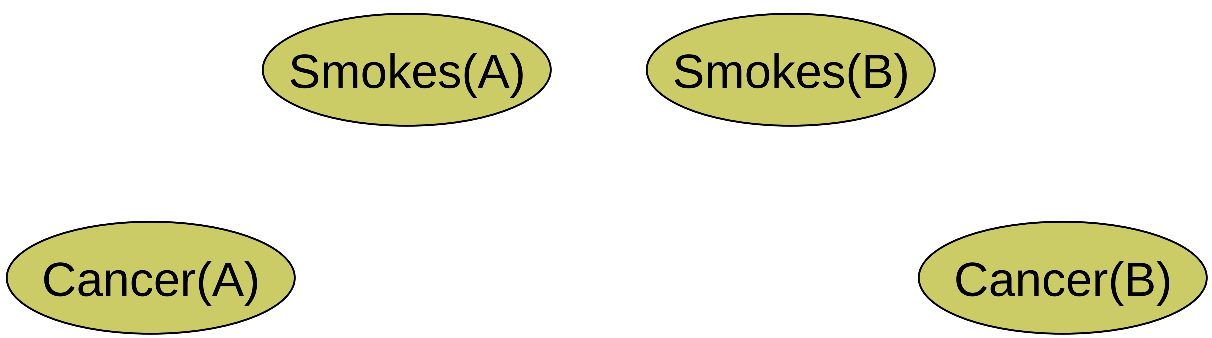
Definition

- A Markov Logic Network (MLN) is a set of pairs (F, w) where
  - F is a formula in first-order logic
  - w is a real number
- Together with a finite set of constants, it defines a Markov network with
  - One node for each grounding of each predicate in the MLN
  - One feature for each grounding of each formula F in the MLN, with the corresponding weight w

Example of an MLN

1.5	$\forall x\ Smokes(x) \Rightarrow Cancer(x)$
1.1	$\forall x, y\ Friends(x, y) \Rightarrow \left(Smokes(x) \Leftrightarrow Smokes(y)\right)$

Suppose we have two constants: **Anna** (A) and **Bob** (B)



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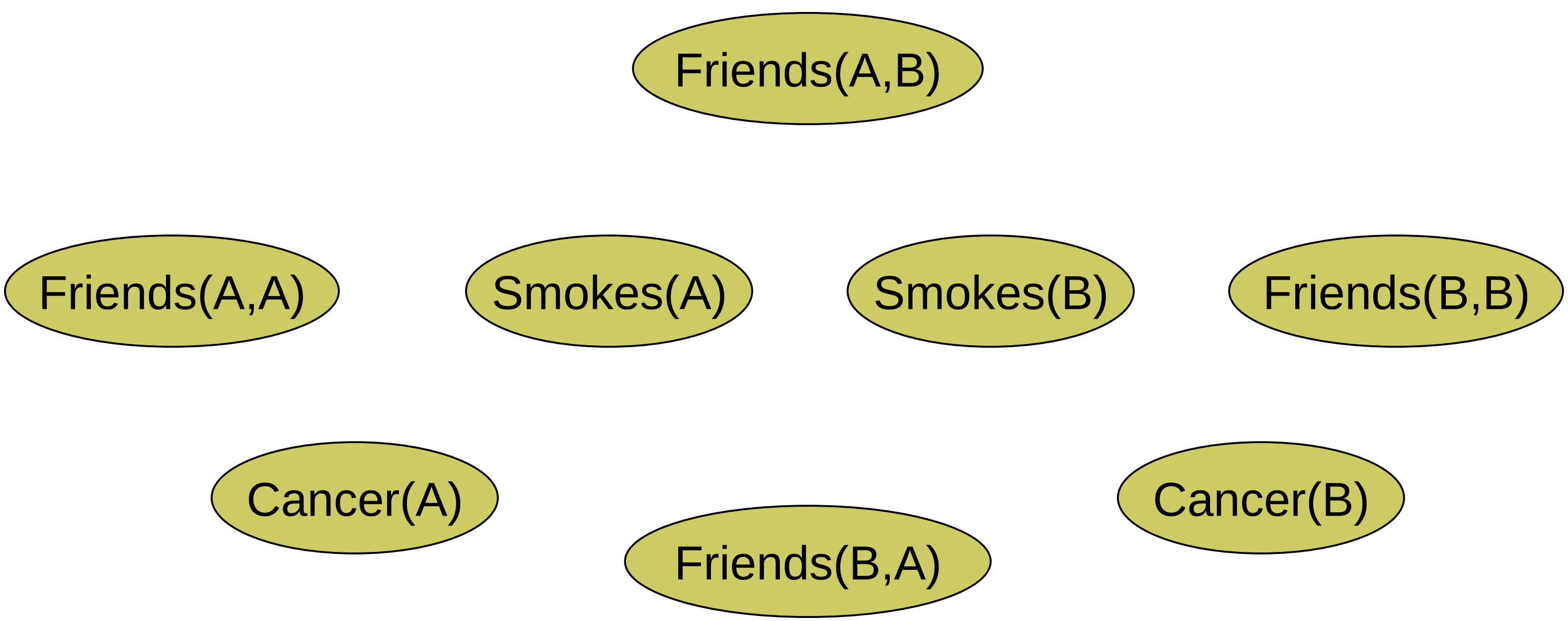

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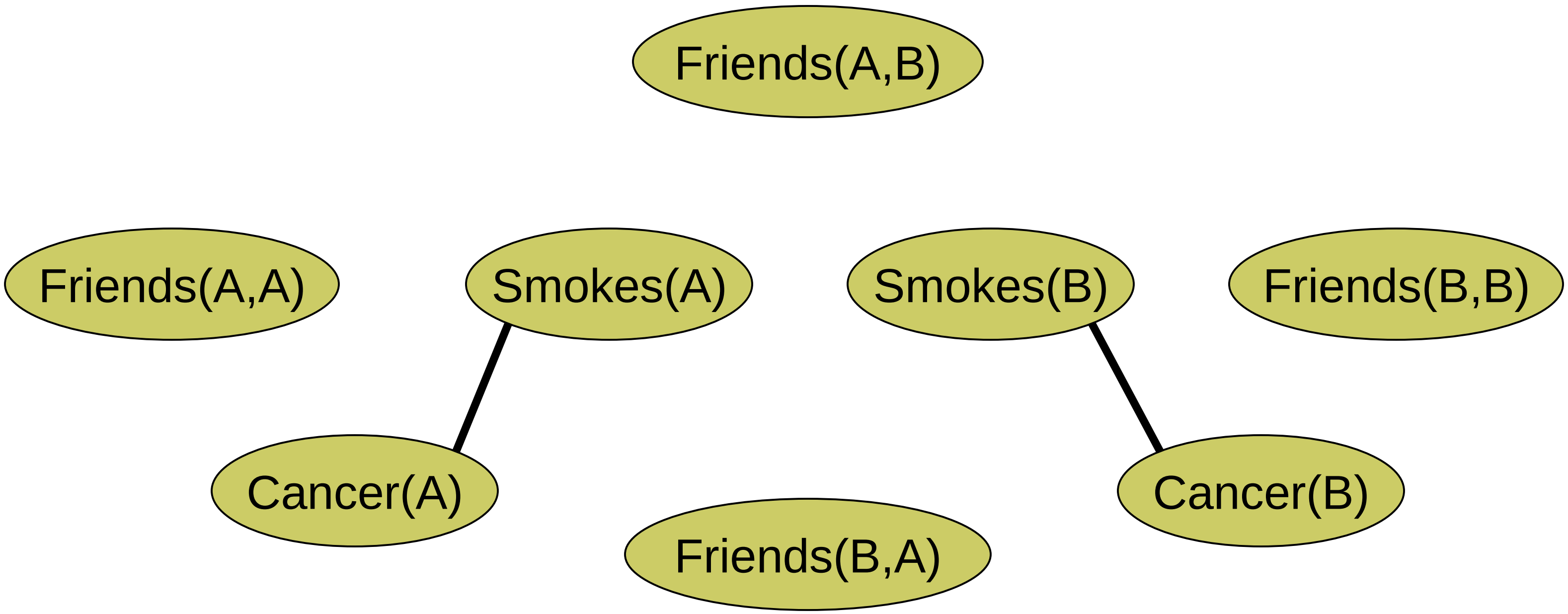

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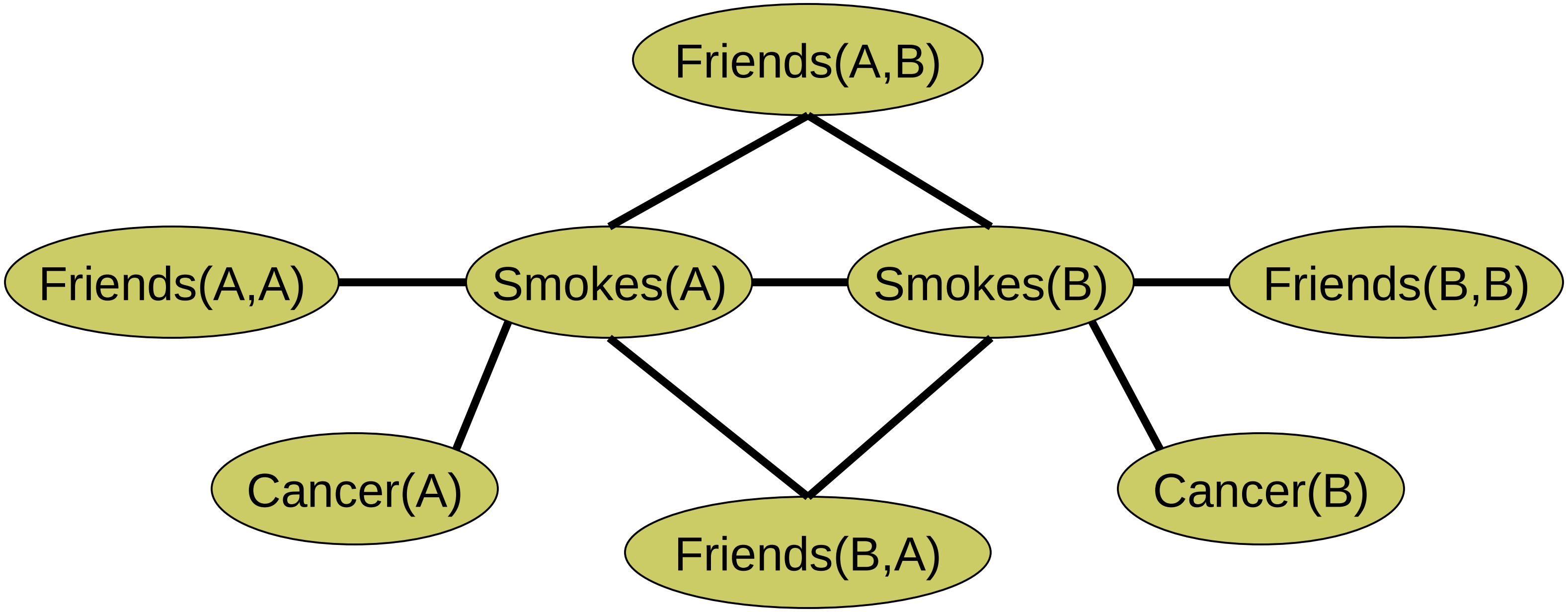

More on MLNs

- **Graph structure:** Arc between two nodes iff predicates appear together in some formula
- MLN is **template** for ground Markov nets
- **Typed** variables and constants greatly reduce size of ground Markov net
- Functions, existential quantifiers, etc.
- MLN without variables = Markov network (subsumes graphical models)

MLNs and First-Order Logic

- Infinite weights $\Rightarrow$ First-order logic
- Satisfiable KB, positive weights $\Rightarrow$ Satisfying assignments = Modes of distribution
- MLNs allow contradictions between formulas
- How to break KB into formulas?
  - Adding probability increases degrees of freedom
  - Knowledge engineering decision
  - Default: Convert to clausal form

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Conditional Inference

- P(Formula|MLN,C) = ?
- MCMC: Sample worlds, check formula holds
- P(Formula1|Formula2,MLN,C) = ?
- If Formula2 = Conjunction of ground atoms
  - First construct min subset of network necessary to answer query (generalization of KBMC)
  - Then apply MCMC

Grounding the Template

- Initialize Markov net to contain all query preds
- For each node in network
  - Add node's Markov blanket to network
  - Remove any evidence nodes
- Repeat until done

Example Grounding

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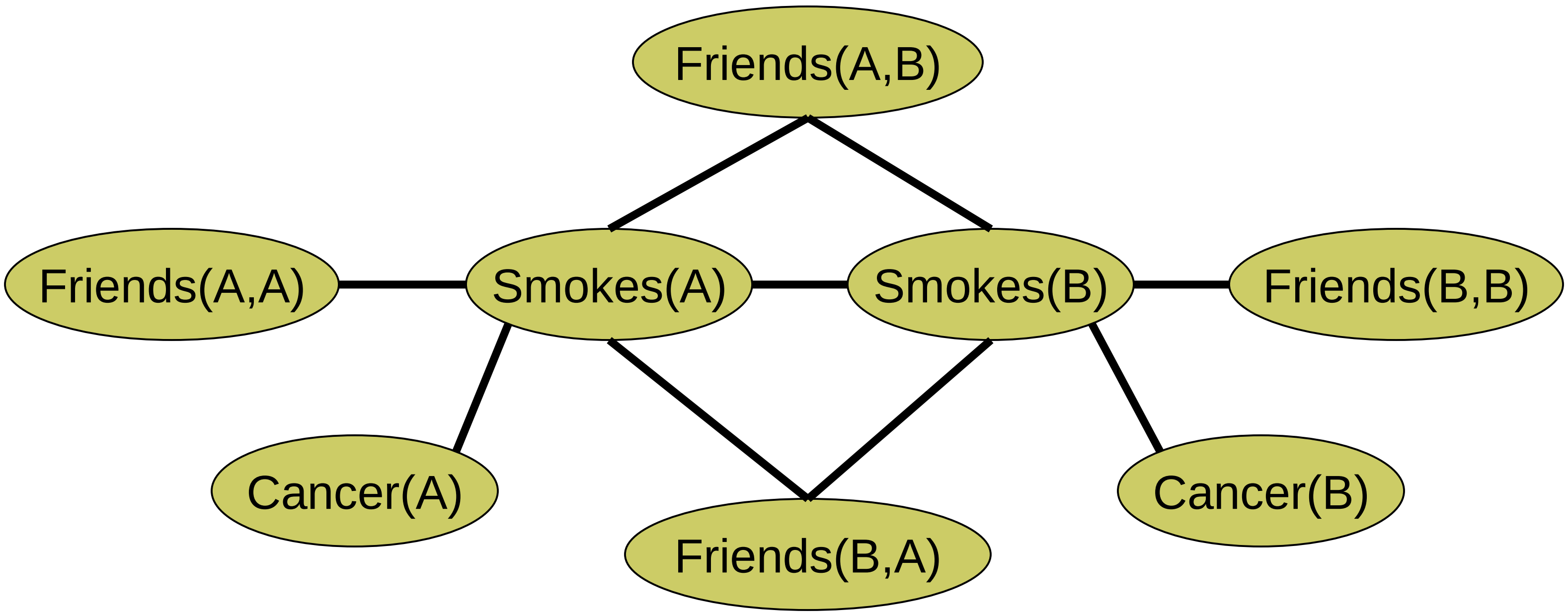

P(Cancer(B) | Smokes(A), Friends(A,B), Friends(B,A))

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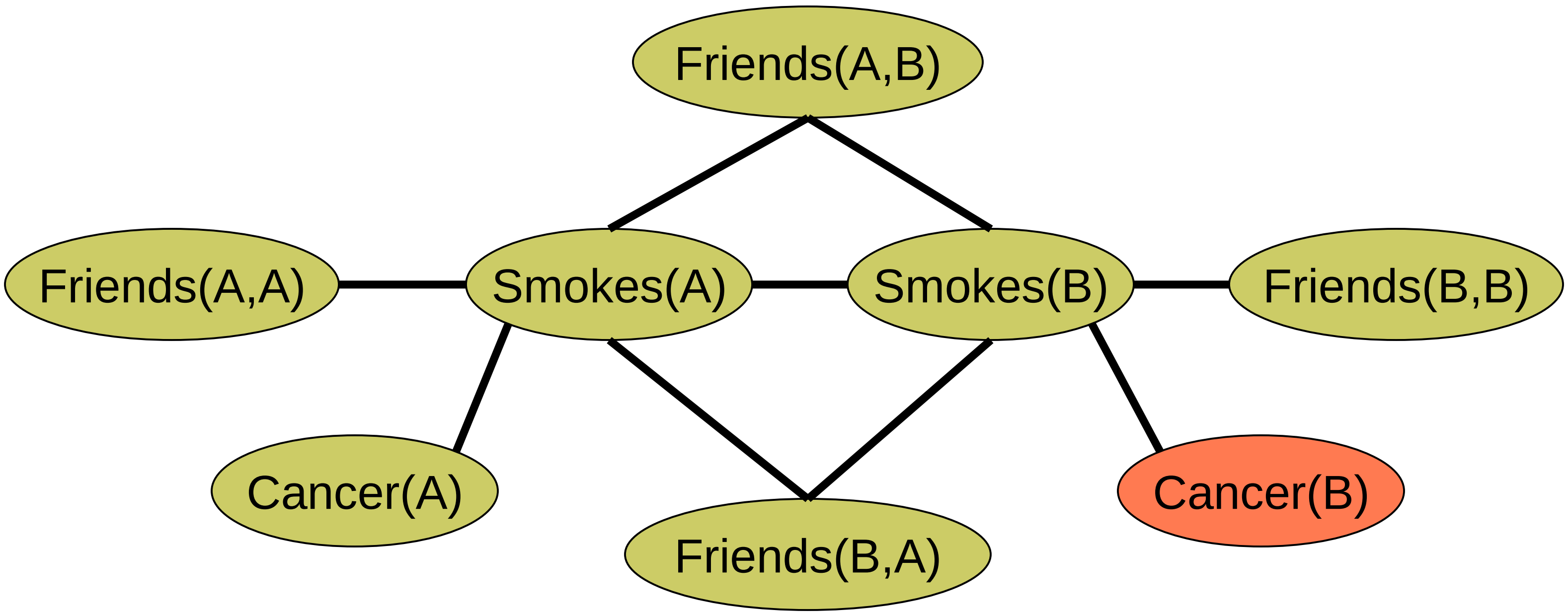

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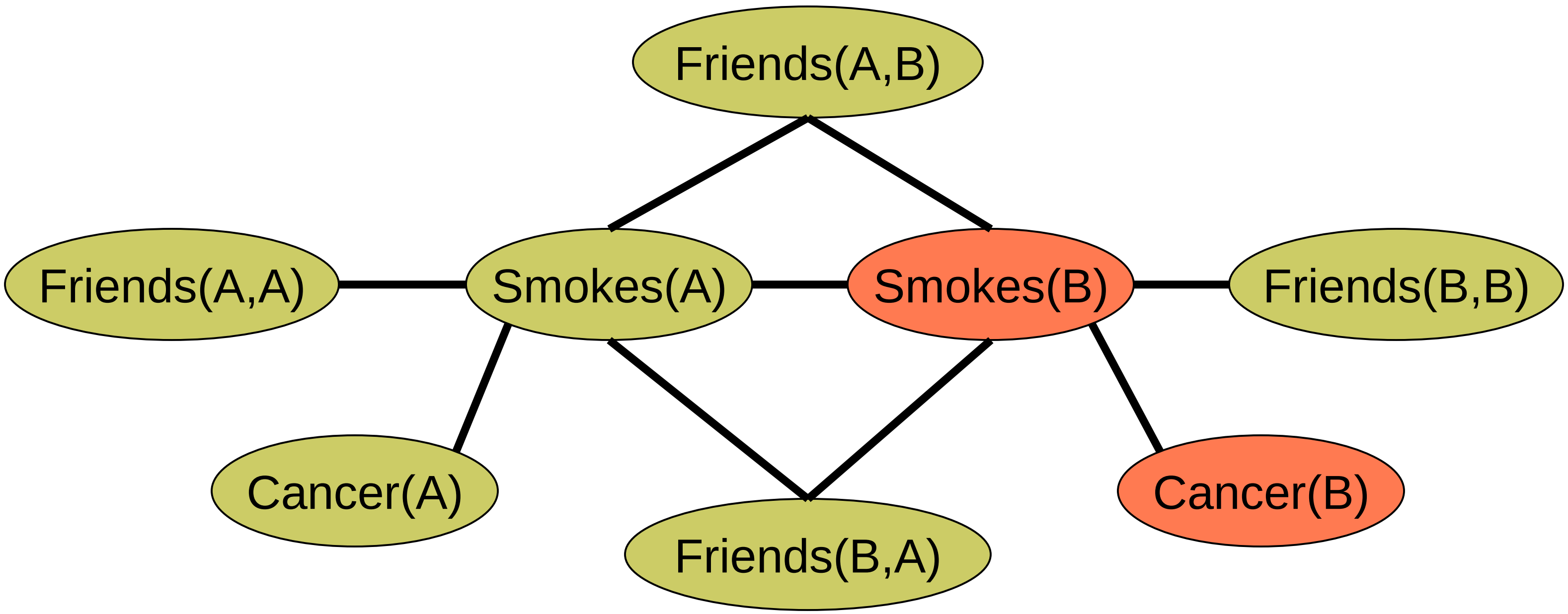

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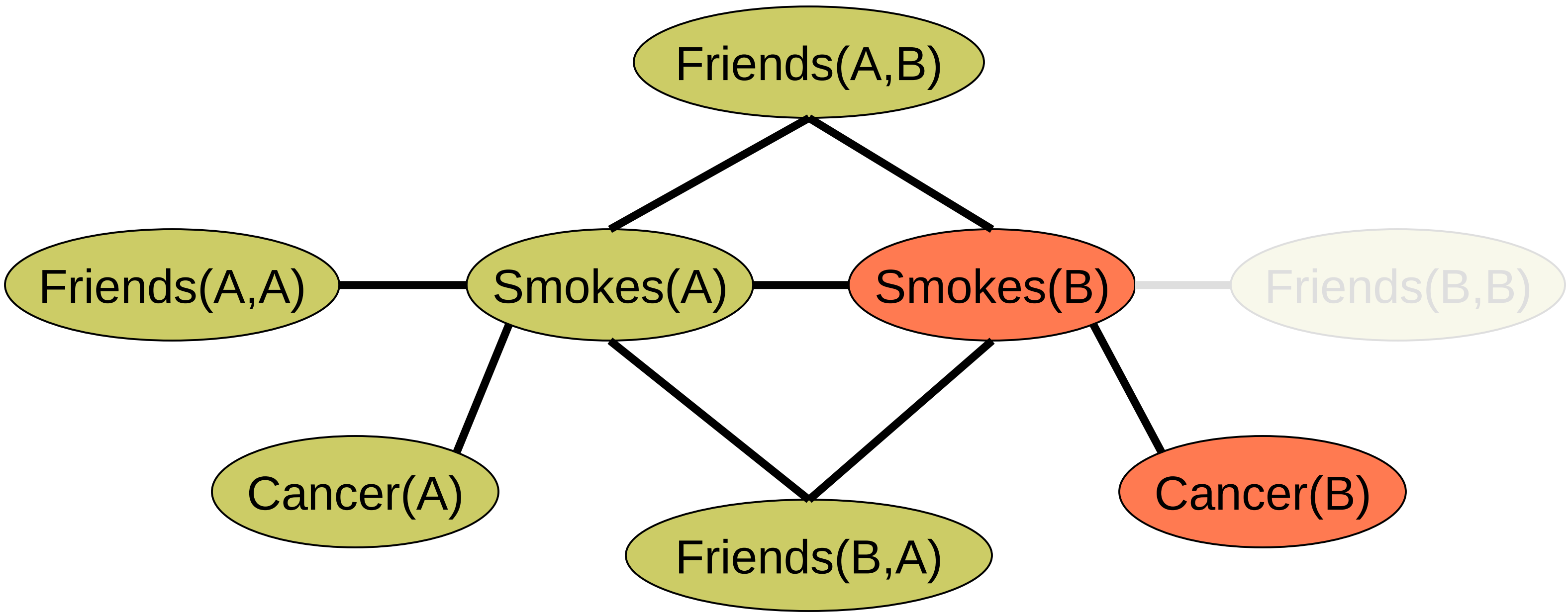

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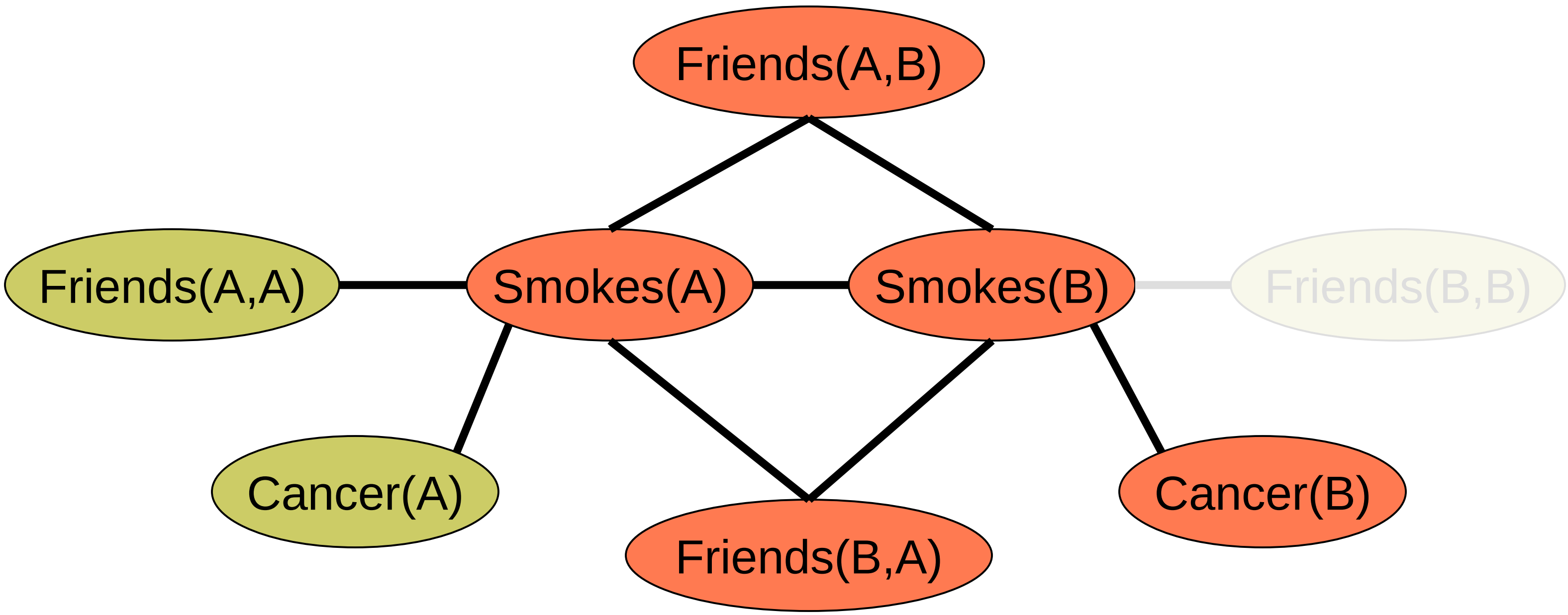

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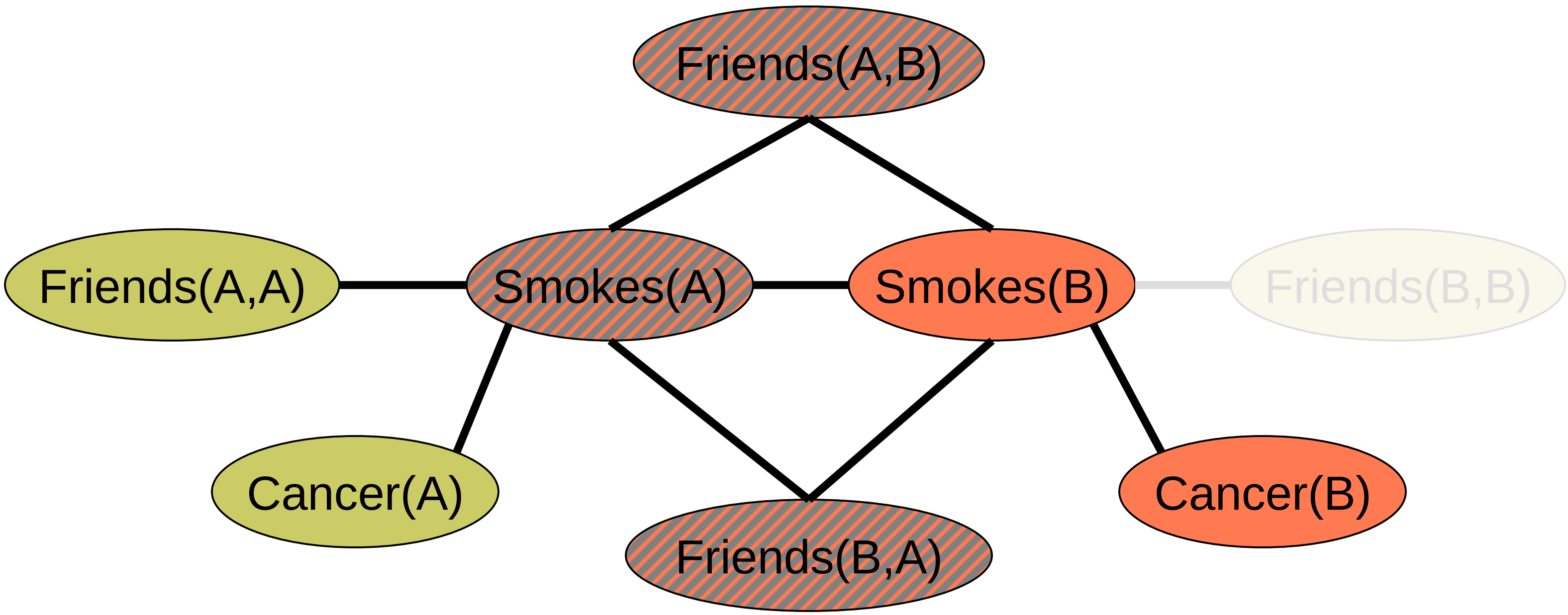

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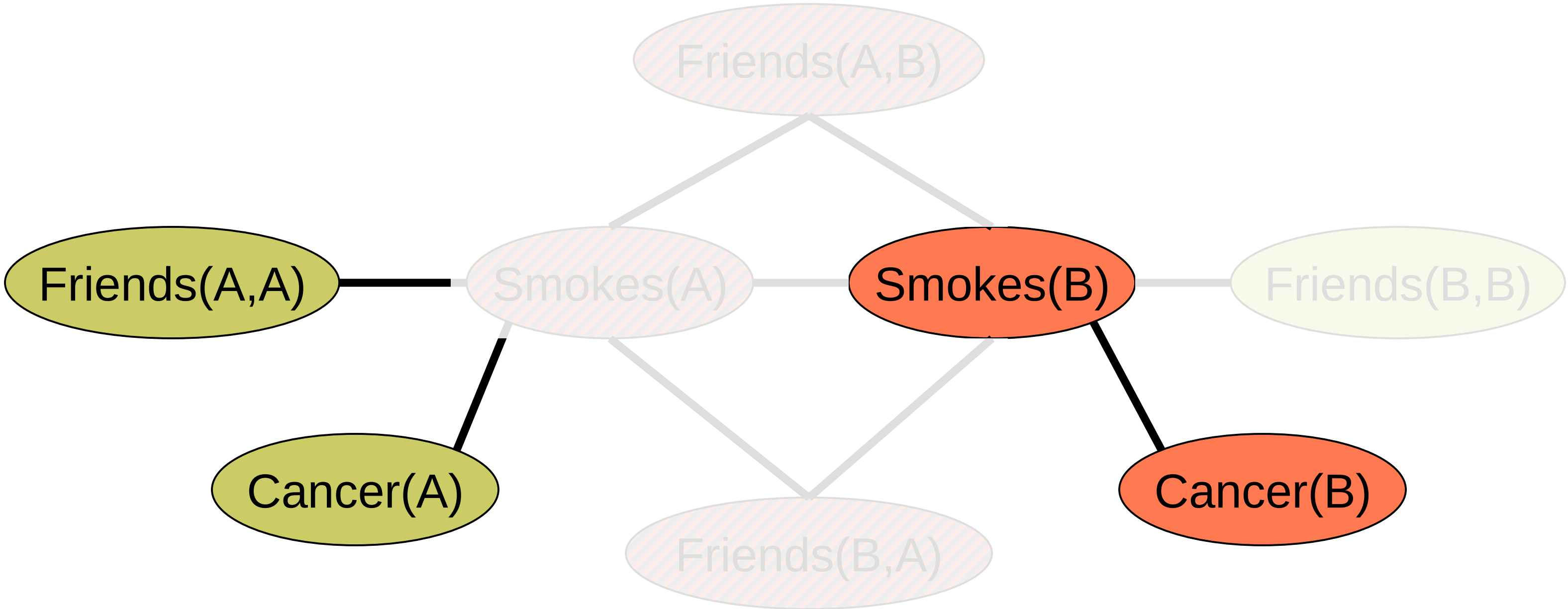

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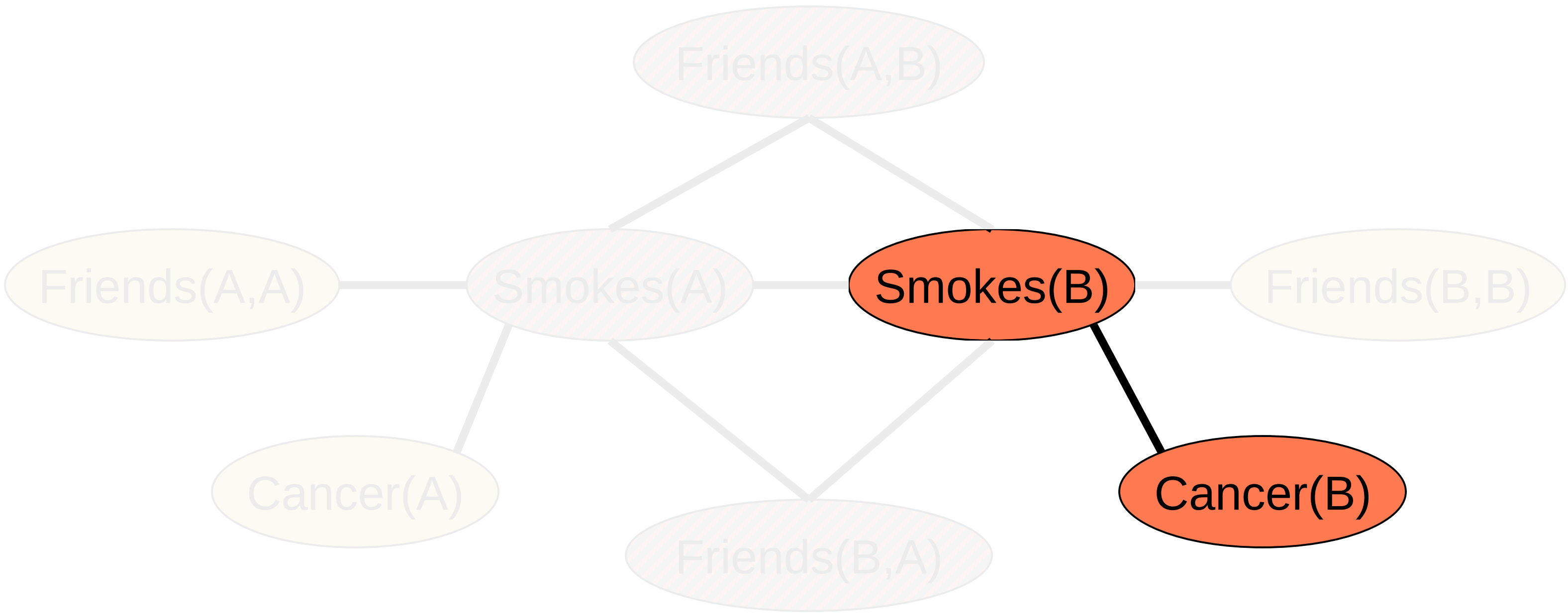

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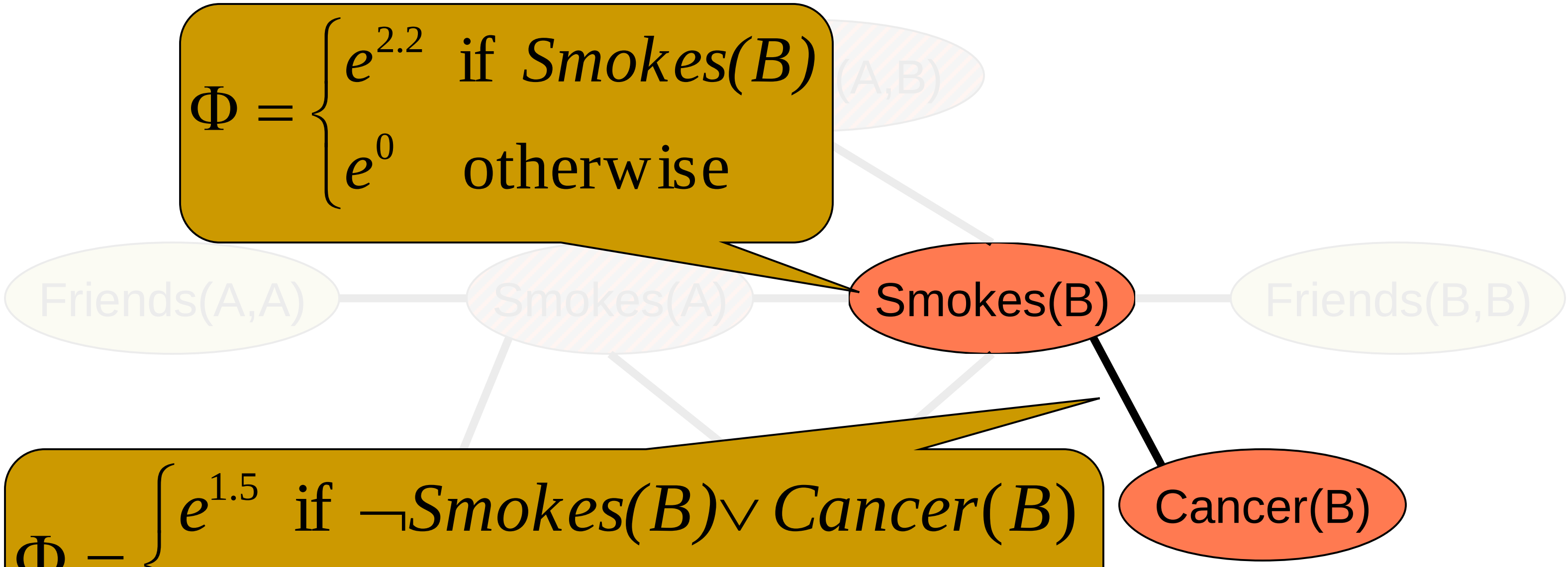
$$\Phi = \begin{cases} e^{2.2} & \text{if } Smokes(B) \\ e^{0} & \text{otherwise} \end{cases}$$



$$\Phi = \begin{cases} e^{1.5} & \text{if } \neg Smokes(B) \vee Cancer(B) \\ e^{0} & \text{otherwise} \end{cases}$$

P(Cancer(B) | Smokes(A), Friends(A,B), Friends(B,A))

Markov Chain Monte Carlo

- **Gibbs Sampler**
  1. Start with an initial assignment to nodes
  2. One node at a time, sample node given others
  3. Repeat
  4. Use samples to compute P(X)
- Apply to ground network
- Many modes $\Rightarrow$ Multiple chains
- Initialization: MaxWalkSat [Kautz et al., 1997]

MPE Inference

- Find most likely truth values of non-evidence ground atoms given evidence
- Apply weighted satisfiability solver (maxes sum of weights of satisfied clauses)
- MaxWalkSat algorithm [Kautz et al., 1997]
  - Start with random truth assignment
  - With prob **p**, flip atom that maxes weight sum; else flip random atom in unsatisfied clause
  - Repeat **n** times
  - Restart **m** times

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Learning

- Data is a relational database
- Closed world assumption
- Learning structure
  - Corresponds to feature induction in Markov nets
  - Learn / modify clauses
  - ILP (e.g., CLAUDIEN [De Raedt & Dehaspe, 1997])
  - Better approach: Stanley will describe
- Learning parameters (weights)

Learning Weights

- Like Markov nets, except with parameter tying over groundings of same formula

$$\frac{d}{dw_i} \log P(X) = \boxed{f_i(X)} - \boxed{E_Y[f_i(Y)]}$$

Feature count according to data

Feature count according to model

- **1st term:** # true groundings of formula in DB
- **2nd term:** inference required, as before (slow!)

Pseudo-Likelihood [Besag, 1975]

$$PL(X) \equiv \prod_x P(x \mid MB(x))$$

- Likelihood of each ground atom given its Markov blanket in the data
- Does not require inference at each step
- Optimized using L-BFGS [Liu & Nocedal, 1989]

Gradient of Pseudo-Log-Likelihood

$$\nabla_i = \sum_x nsat_i(x) - \left[p(x{=}0)\, nsat_i(x{=}0) + p(x{=}1)\, nsat_i(x{=}1) \right]$$

where $nsat_i(x=v)$ is the number of satisfied groundings of clause i **in the training data** when x takes value v

- Most terms not affected by changes in weights
- After initial setup, each iteration takes O(# ground predicates x # first-order clauses)

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Domain

- University of Washington CSE Dept.
- 12 first-order predicates:
  Professor, Student, TaughtBy, AuthorOf, AdvisedBy, etc.
- 2707 constants divided into 10 types:
  Person (442), Course (176), Pub. (342), Quarter (20), etc.
- 4.1 million ground predicates
- 3380 ground predicates (tuples in database)

Systems Compared

- Hand-built knowledge base (KB)
- ILP: CLAUDIEN [De Raedt & Dehaspe, 1997]
- Markov logic networks (MLNs)
  - Using KB
  - Using CLAUDIEN
  - Using KB + CLAUDIEN
- Bayesian network learner [Heckerman et al., 1995]
- Naïve Bayes [Domingos & Pazzani, 1997]

Sample Clauses in KB

- Students are not professors
- Each student has only one advisor
- If a student is an author of a paper, so is her advisor
- Advanced students only TA courses taught by their advisors
- At most one author of a given paper is a professor

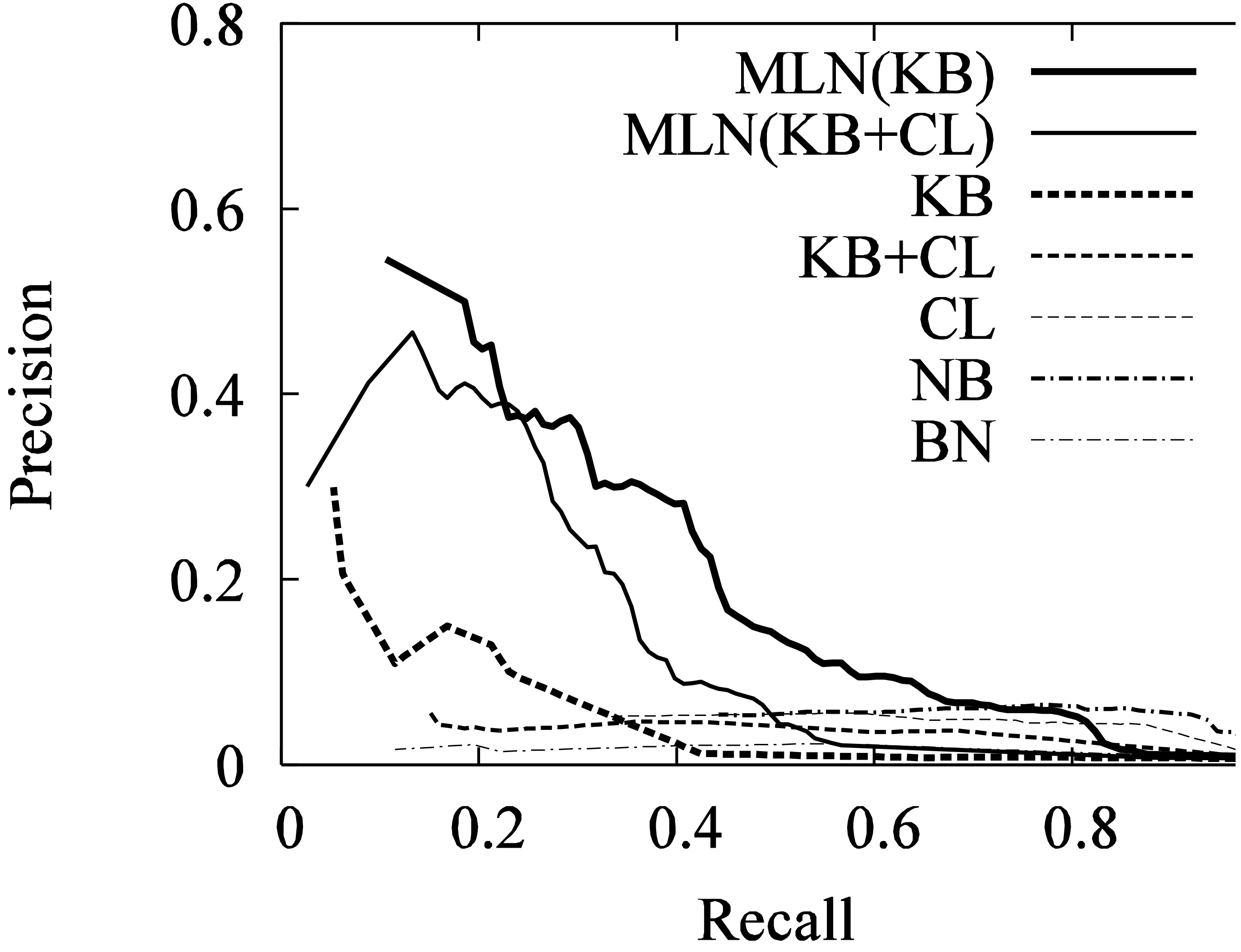
Methodology

- Data split into five areas:
  AI, graphics, languages, systems, theory
- Leave-one-area-out testing
- Task: Predict AdvisedBy(x, y)
  - All Info: Given all other predicates
  - Partial Info: With Student(x) and Professor(x) missing
- Evaluation measures:
  - Conditional log-likelihood
    (KB, CLAUDIEN: Run WalkSat 100x to get probabilities)
  - Area under precision-recall curve

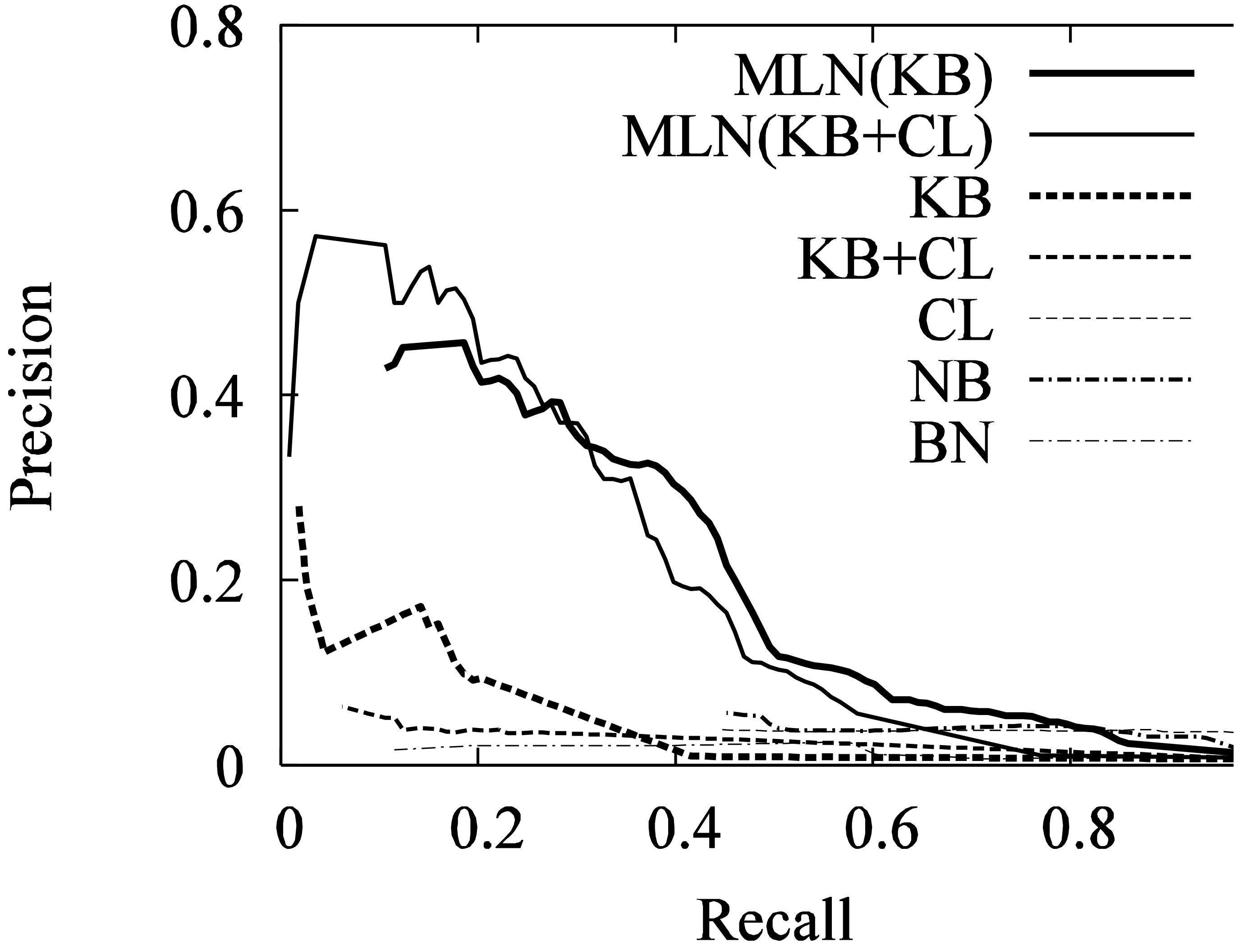
Results

System	All Info		Partial Info	
	CLL	AUC	CLL	AUC
MLN(KB+CL)	-0.058	0.152	-0.045	0.203
MLN(KB)	-0.052	0.215	-0.048	0.224
MLN(CL)	-2.315	0.035	-2.478	0.032
KB	-0.135	0.059	-0.063	0.048
CL	-0.434	0.048	-0.836	0.037
NB	-1.214	0.054	-1.140	0.044
BN	-0.072	0.015	-0.215	0.015

Results: All Info



Results: Partial Info



Efficiency

- Learning time: 16 mins
- Time to infer all AdvisedBy predicates:
  - With complete info: 8 mins
  - With partial info: 15 mins

(124K Gibbs passes)

Other Applications

- UW-CSE task: Link prediction
- Collective classification
- Link-based clustering
- Social network models
- Object identification
- Etc.

Other SRL Approaches are Special Cases of MLNs

- Probabilistic relational models
  (Friedman et al, IJCAI-99)
- Stochastic logic programs
  (Muggleton, SRL-00)
- Bayesian logic programs
  (Kersting & De Raedt, ILP-01)
- Relational Markov networks
  (Taskar et al, UAI-02)
- Etc.

Open Problems: Inference

- Lifted inference
- Better MCMC (e.g., Swendsen-Wang)
- Belief propagation
- Selective grounding
- Abstraction, summarization, multi-scale
- Special cases

Open Problems: Learning

- Discriminative training
- Learning and refining structure
- Learning with missing info
- Faster optimization
- Beyond pseudo-likelihood
- Learning by reformulation

Open Problems: Applications

- Information extraction & integration
- Semantic Web
- Social networks
- Activity recognition
- Parsing with world knowledge
- Scene analysis with world knowledge
- Etc.

Summary

- **Markov logic networks** combine first-order logic and Markov networks
  - **Syntax:** First-order logic + Weights
  - **Semantics:** Templates for Markov networks
- **Inference:** KBMC + MaxWalkSat + MCMC
- **Learning:** ILP + Pseudo-likelihood
- SRL problems easily formulated as MLNs
- Many open research issues