Linear Gaussian Systems

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- Suppose we have two variables $\mathbf{x} \in \mathbb{R}^{D_x}$, $\mathbf{y} \in \mathbb{R}^{D_y}$ where $\mathbf{y}$ is a noisy observation of $\mathbf{x}$, with the following prior and likelihood:

$$\begin{aligned} p(\mathbf{x}) &= \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_x, \boldsymbol{\Sigma}_x) \\ p(\mathbf{y}|\mathbf{x}) &= \mathcal{N}(\mathbf{y}|\mathbf{A}\mathbf{x} + \mathbf{b}, \boldsymbol{\Sigma}_y) \end{aligned}$$

Bayes rule for Linear Gaussian Systems

- Given a linear Gaussian system, the posterior $p(\mathbf{x}|\mathbf{y})$ is given as follows:

$$
\begin{aligned}
p(\mathbf{x}|\mathbf{y}) &= \mathcal{N}\left(\mathbf{x}|\boldsymbol{\mu}_{x|y}, \boldsymbol{\Sigma}_{x|y}\right) \\
{\boldsymbol{\Sigma}_{x|y}}^{-1} &= {\boldsymbol{\Sigma}_x}^{-1} + \mathbf{A}^T{\boldsymbol{\Sigma}_y}^{-1}\mathbf{A} \\
\boldsymbol{\mu}_{x|y} &= \boldsymbol{\Sigma}_{x|y}\left[\mathbf{A}^T{\boldsymbol{\Sigma}_y}^{-1}(\mathbf{y}-\mathbf{b}) + {\boldsymbol{\Sigma}_x}^{-1}\boldsymbol{\mu}_x\right]
\end{aligned}
$$

- In addition, the normalization constant $p(\mathbf{y})$ is given by:

$$
p(\mathbf{y}) = \mathcal{N}\left(\mathbf{y}|\mathbf{A}\mu_x + \mathbf{b}, \mathbf{A}\boldsymbol{\Sigma}_x\mathbf{A}^T\right)
$$