6.001 SICP
Environment model

- Models of computation
  - Substitution model
    - A way to figure out what happens during evaluation
      - `(define l '(a b c))`
      - `(car l)` ➔ `a`
      - `(define m '(1 2 3))`
      - `(car l)` ➔ `a`
    - Not really what happens in the computer
      - `(car l)` ➔ `a`
      - `(set-car! l 'z)`
      - `(car l)` ➔ `z`
  - The Environment Model

Can you figure out why this code works?

```
(define make-counter
   (lambda (n)
     (lambda () (set! n (+ n 1))
                n   )))

 (define ca (make-counter 0))
 (ca) ==> 1
 (ca) ==> 2    ; not functional programming!
 (define cb (make-counter 0))
 (cb) ==> 1
 (ca) ==> 3    ; ca and cb are independent
```

What the EM is:

- A precise, completely mechanical description of:
  - name-rule looking up the value of a variable
  - define-rule creating a new definition of a var
  - set!-rule changing the value of a variable
  - lambda-rule creating a procedure
  - application applying a procedure

- Enables analyzing more complex scheme code:
  - Example: **make-counter**

- Basis for implementing a scheme interpreter
  - for now: draw EM state with boxes and pointers
  - later on: implement with code

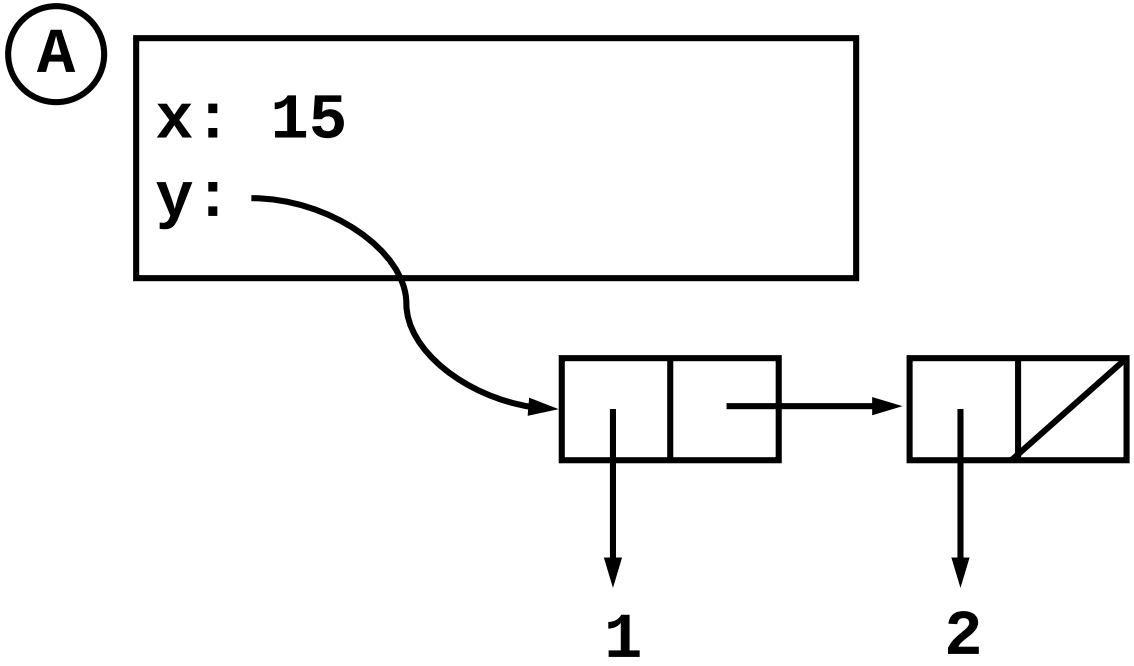
A shift in viewpoint

- As we introduce the environment model, we are going to shift our viewpoint on computation
- Variable:
  - OLD – name for value
  - NEW – place into which one can store things
- Procedure:
  - OLD – functional description
  - NEW – object with inherited context
- Expressions
  - Now **only** have meaning with respect to an environment

Frame: a table of bindings

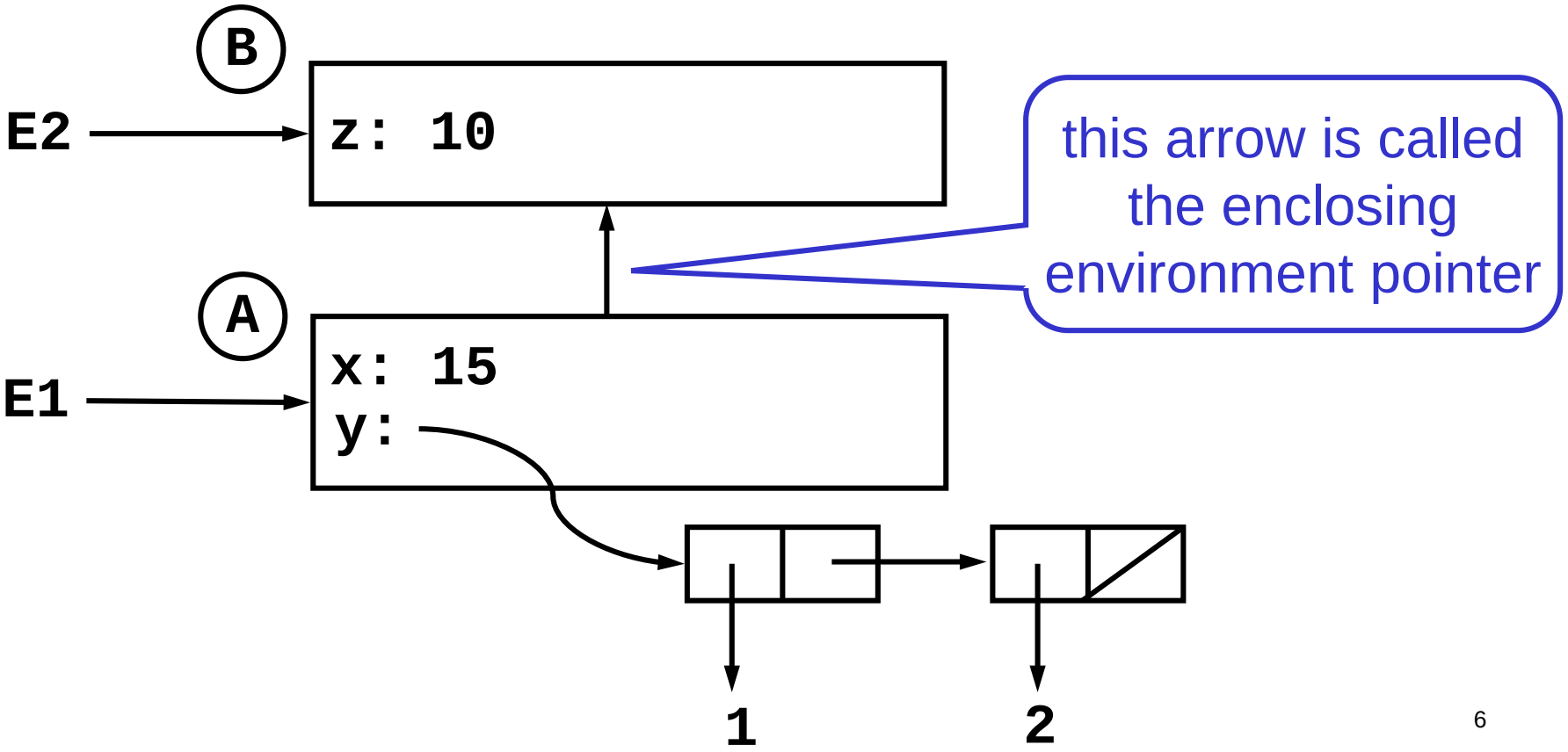
- Binding: a pairing of a name and a value

Example: **x** is bound to **15** in frame A
y is bound to **(1 2)** in frame A
the value of the variable **x** in frame A is 15



Environment: a sequence of frames

- Environment E1 consists of frames A and B
- Environment E2 consists of frame B only
  - A frame may be shared by multiple environments



Evaluation in the environment model

- All evaluation occurs in an environment
  - The current environment changes when the interpreter applies a procedure

- The top environment is called the global environment (GE)
  - Only the GE has no enclosing environment

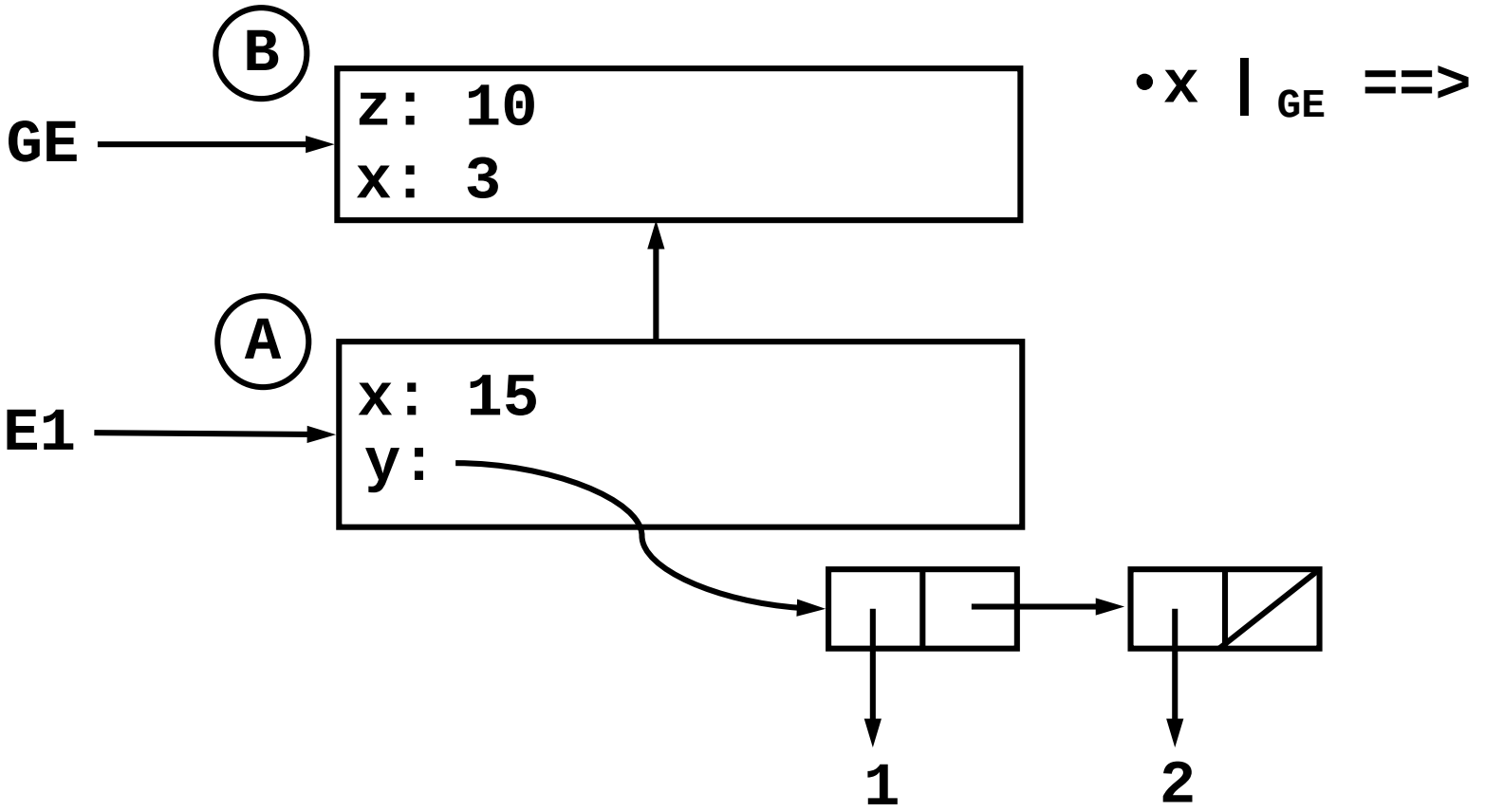
- To evaluate a combination
  - Evaluate the subexpressions *in the current environment*
  - Apply the value of the first to the values of the rest

Name-rule

- A name X evaluated in environment E gives
  the value of X in *the first frame of E where X is bound*

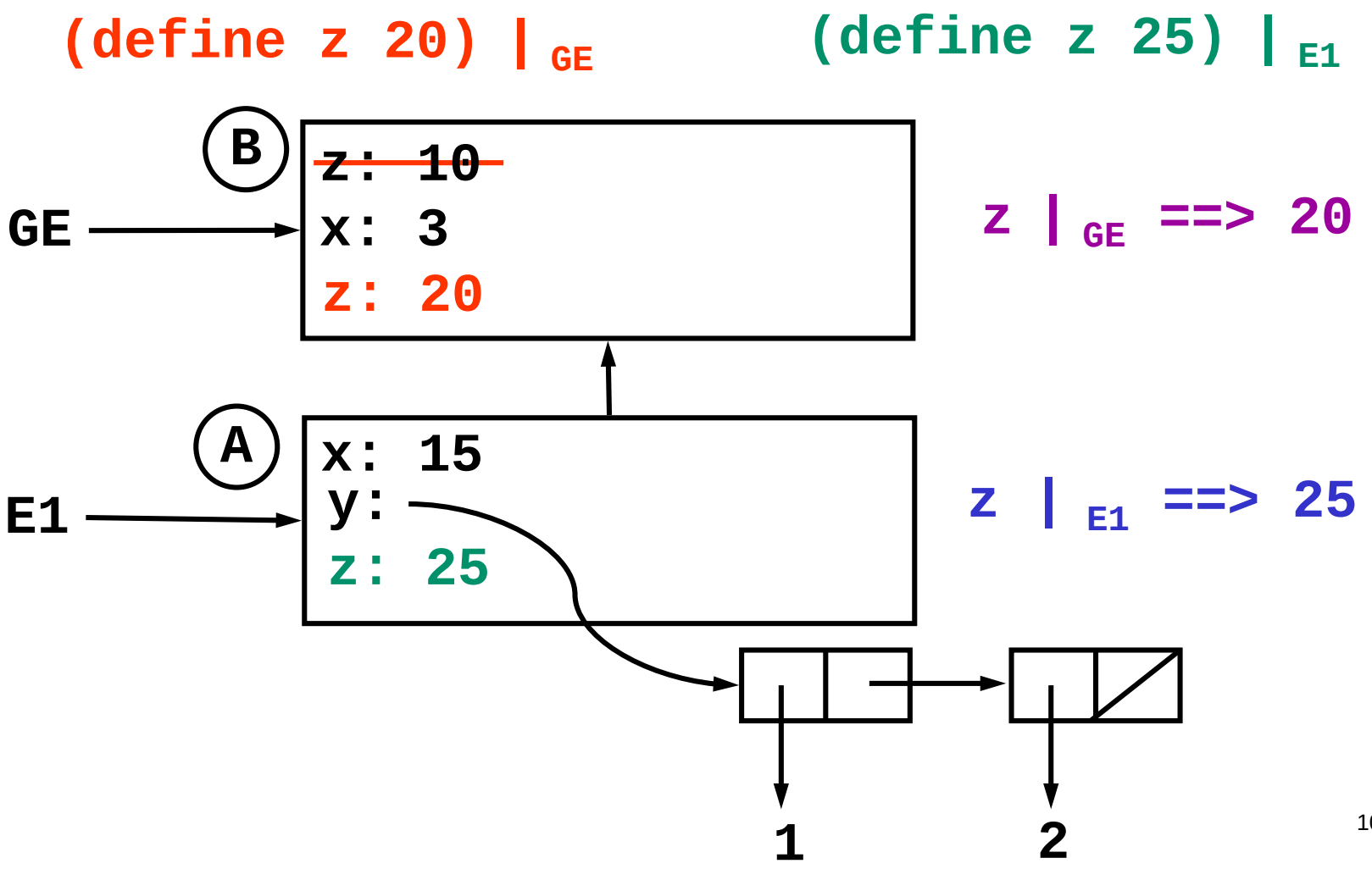
•`z | GE ==>` `z | E1 ==>` `x | E1 ==>`

- In E1, the binding of x in frame A shadows the binding of x in B



Define-rule

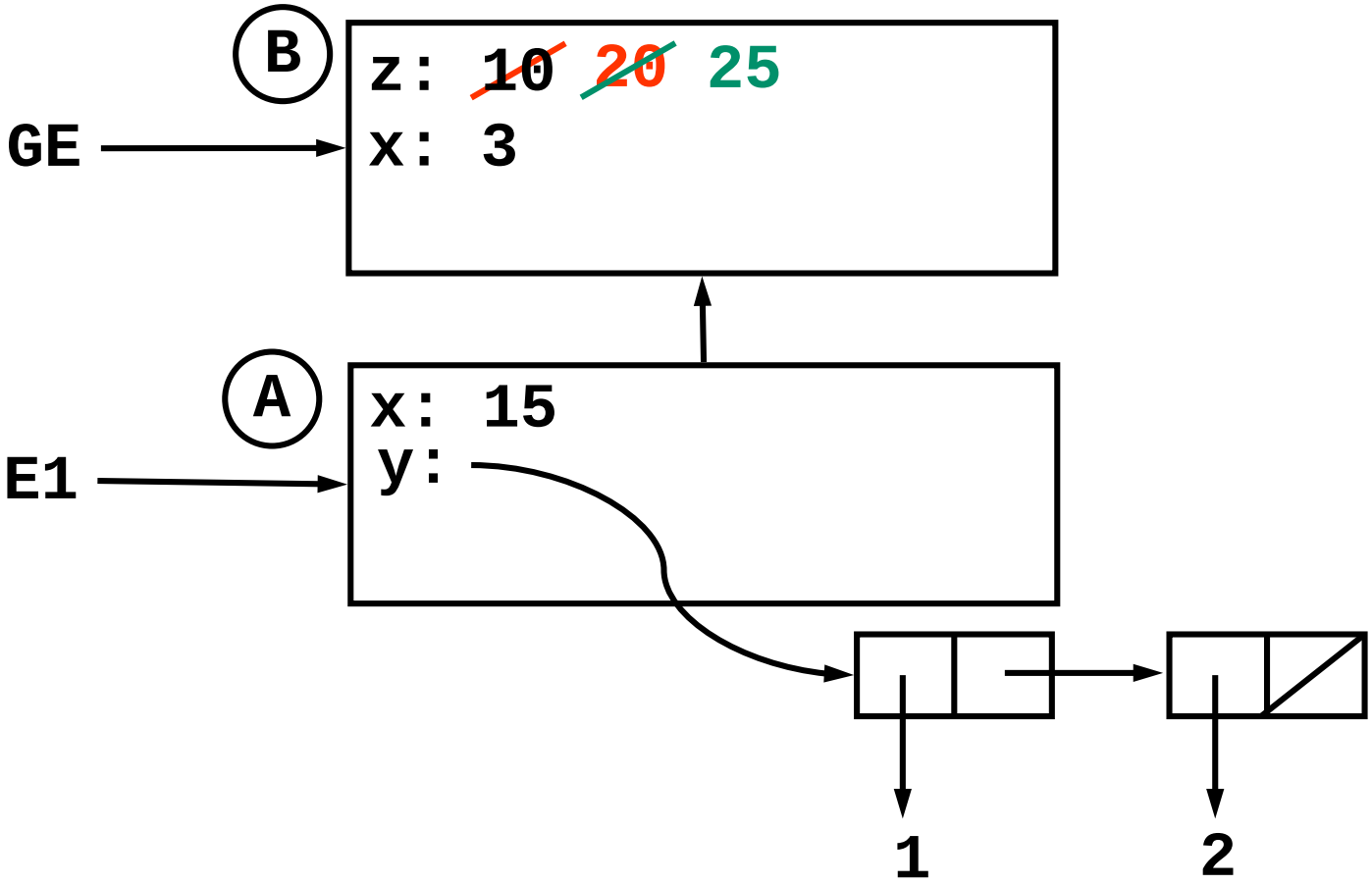
- A define special form evaluated in environment E creates or replaces a binding in the first frame of E



Set!-rule

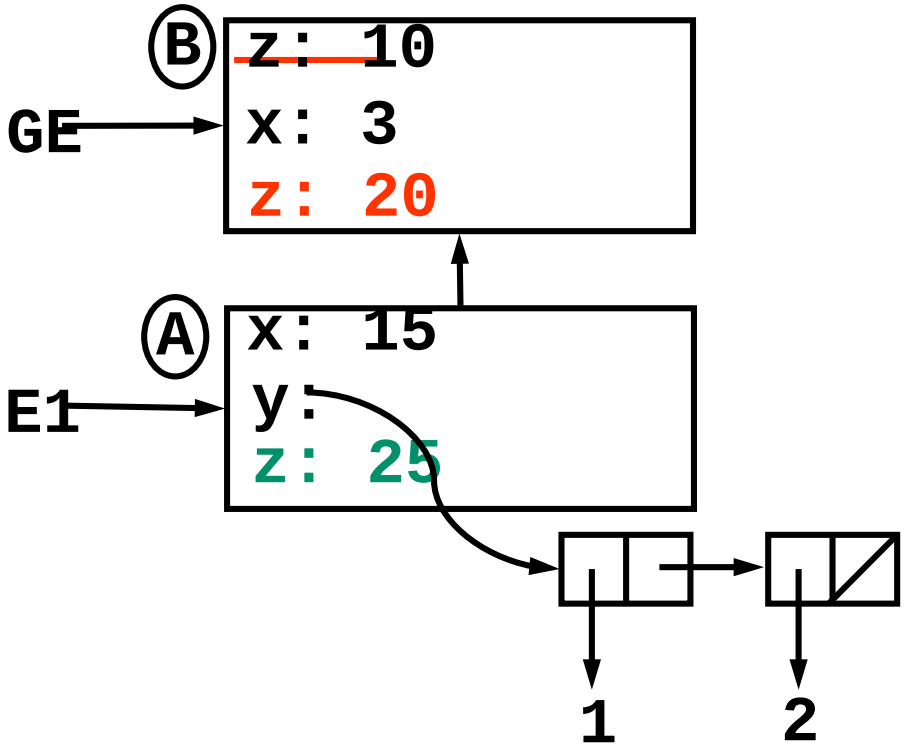
- A set! of variable X evaluated in environment E changes the binding of X in the first frame of E where X is bound

`(set! z 20)` | GE `(set! z 25)` | E1

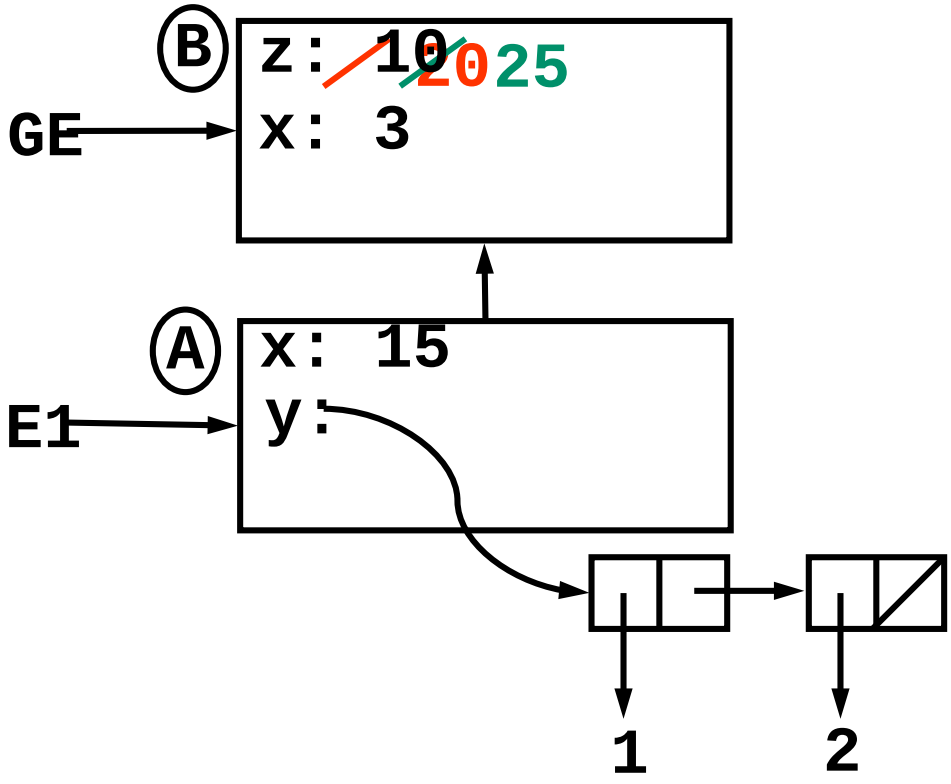


Define versus Set!

Using defines



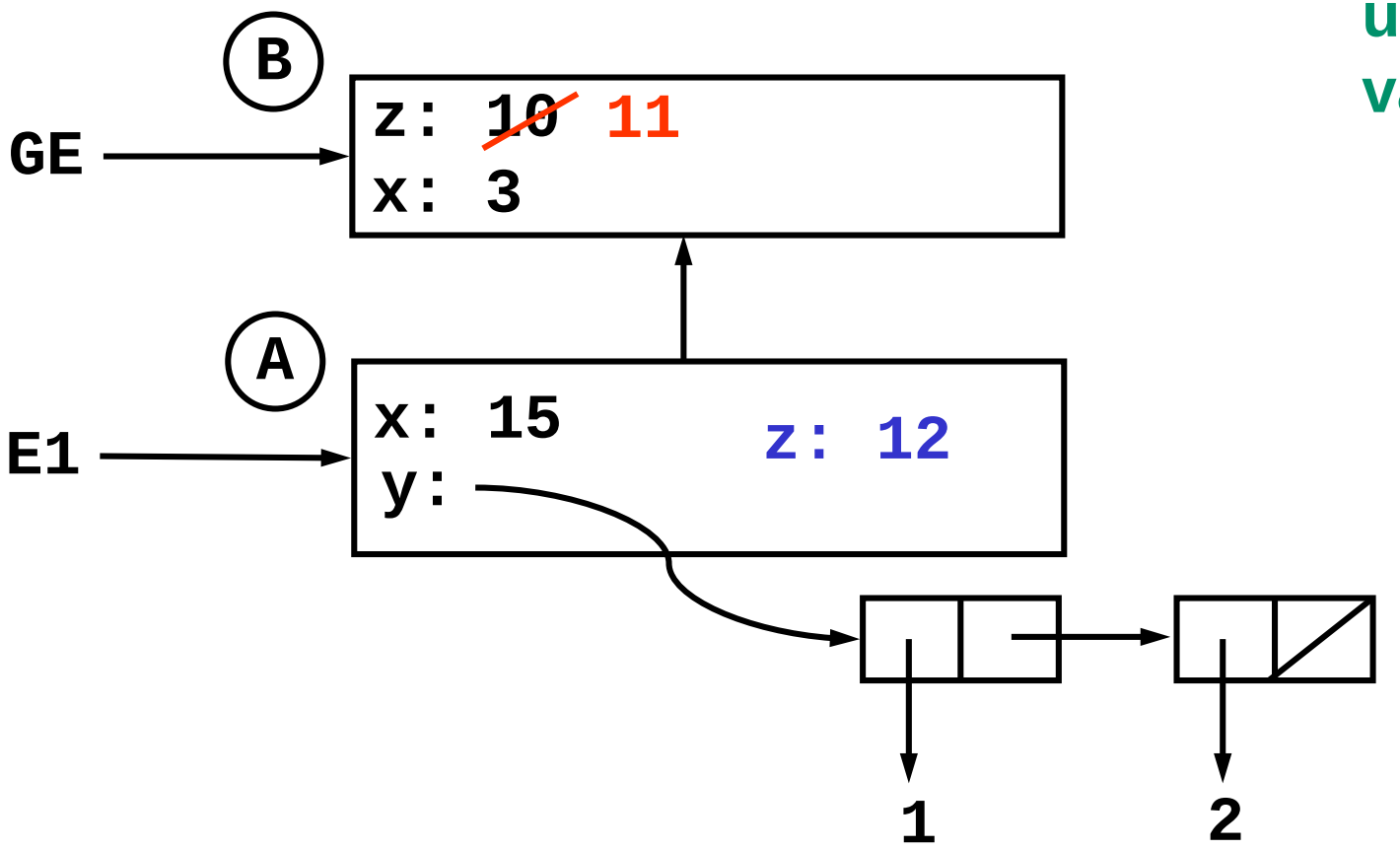
Using set!s



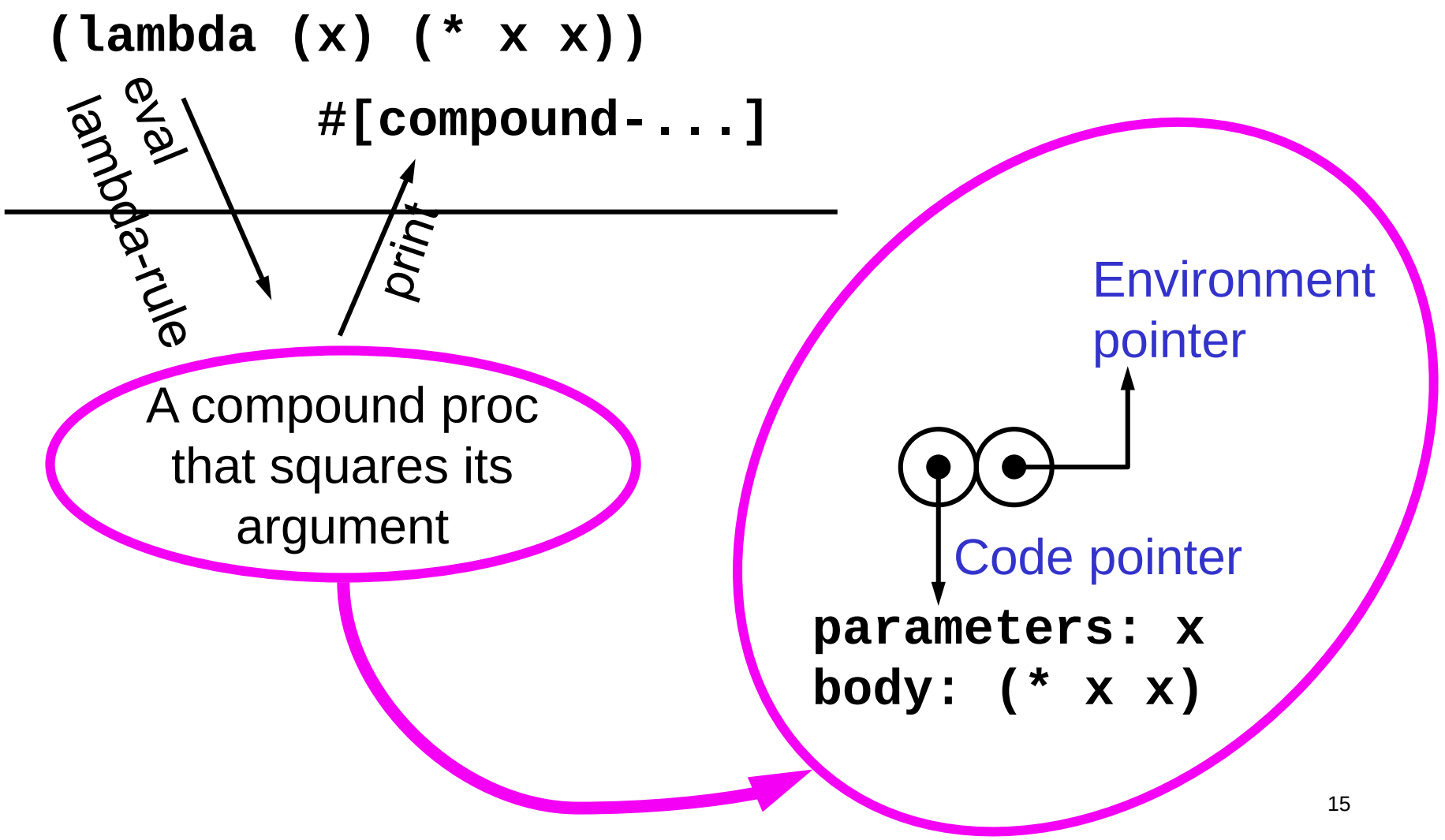
Your turn: evaluate the following in order

```
(+ z 1) |E1                    ==>   11
(set! z (+ z 1)) |E1           (modify EM)
(define z (+ z 1)) |E1         (modify EM)
(set! y (+ z 1)) |GE           (modify EM)
```

Error: unbound variable: y



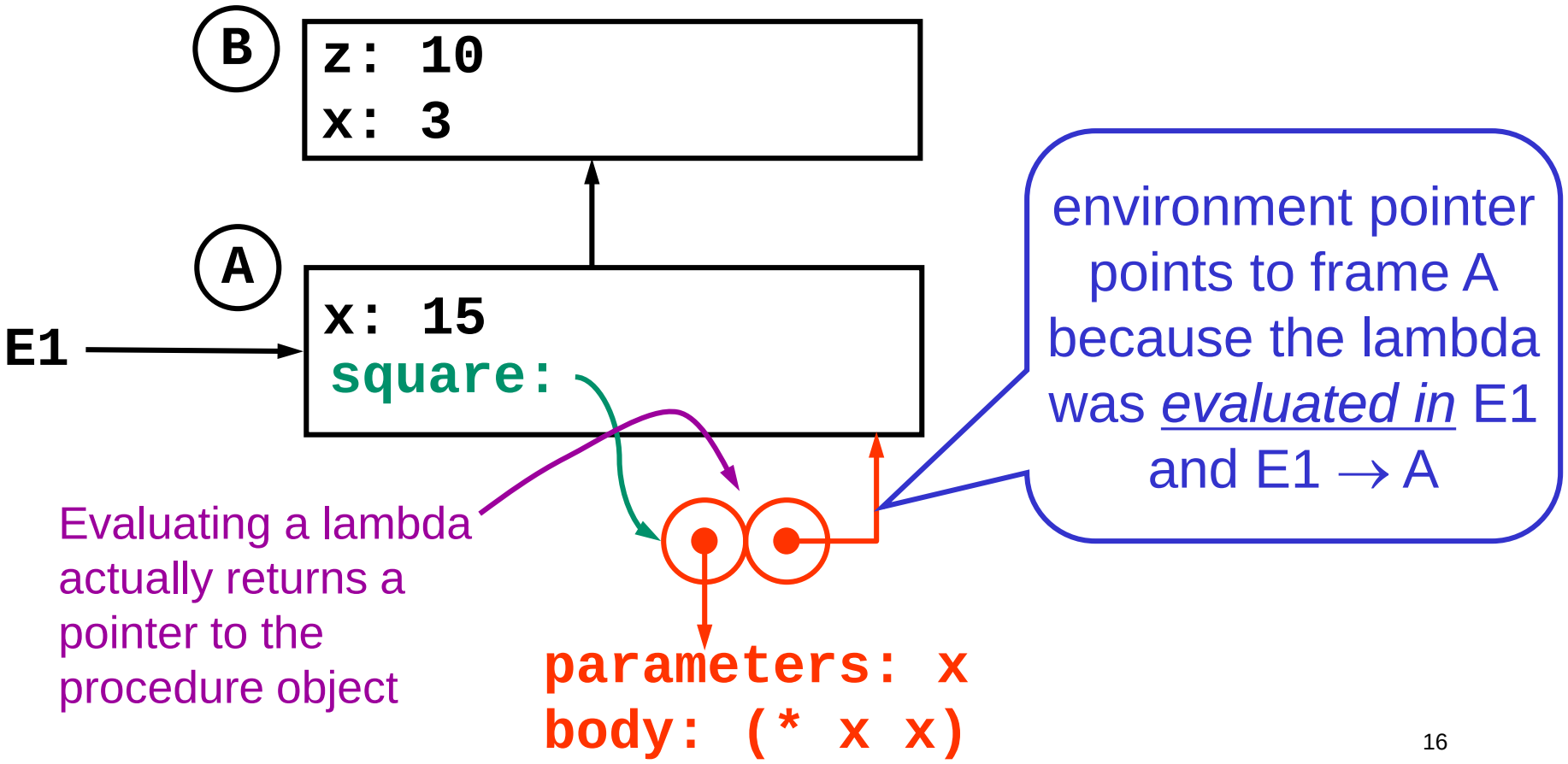
Double bubble: how to draw a procedure



Lambda-rule

- A lambda special form evaluated in environment E creates a procedure whose environment pointer is E

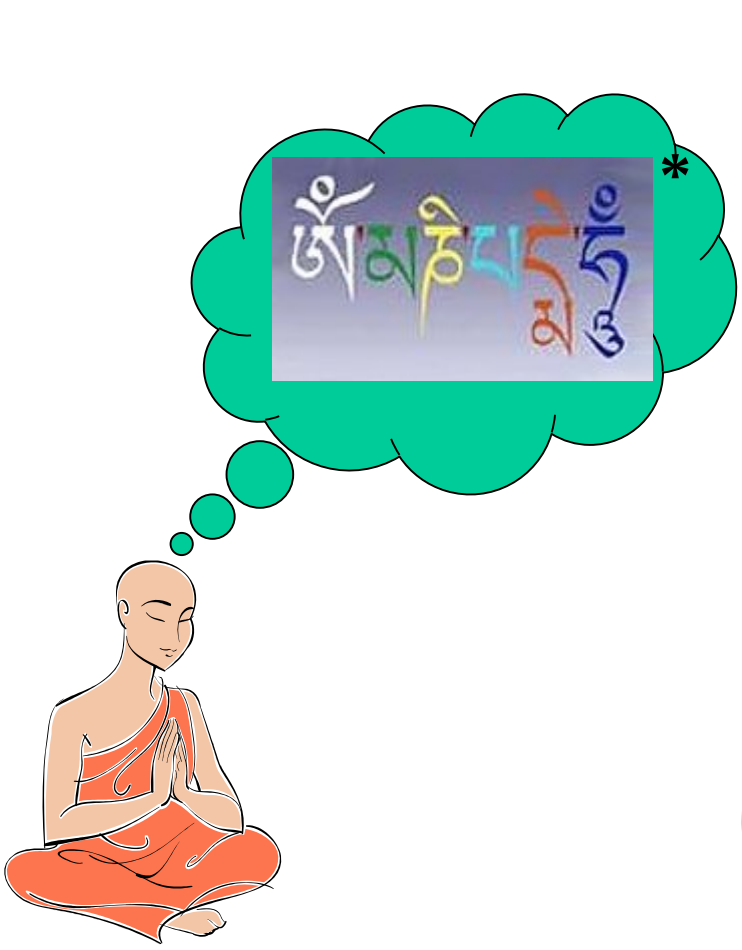
`(define square (lambda (x) (* x x)))` | E1



To apply a compound procedure P to arguments:

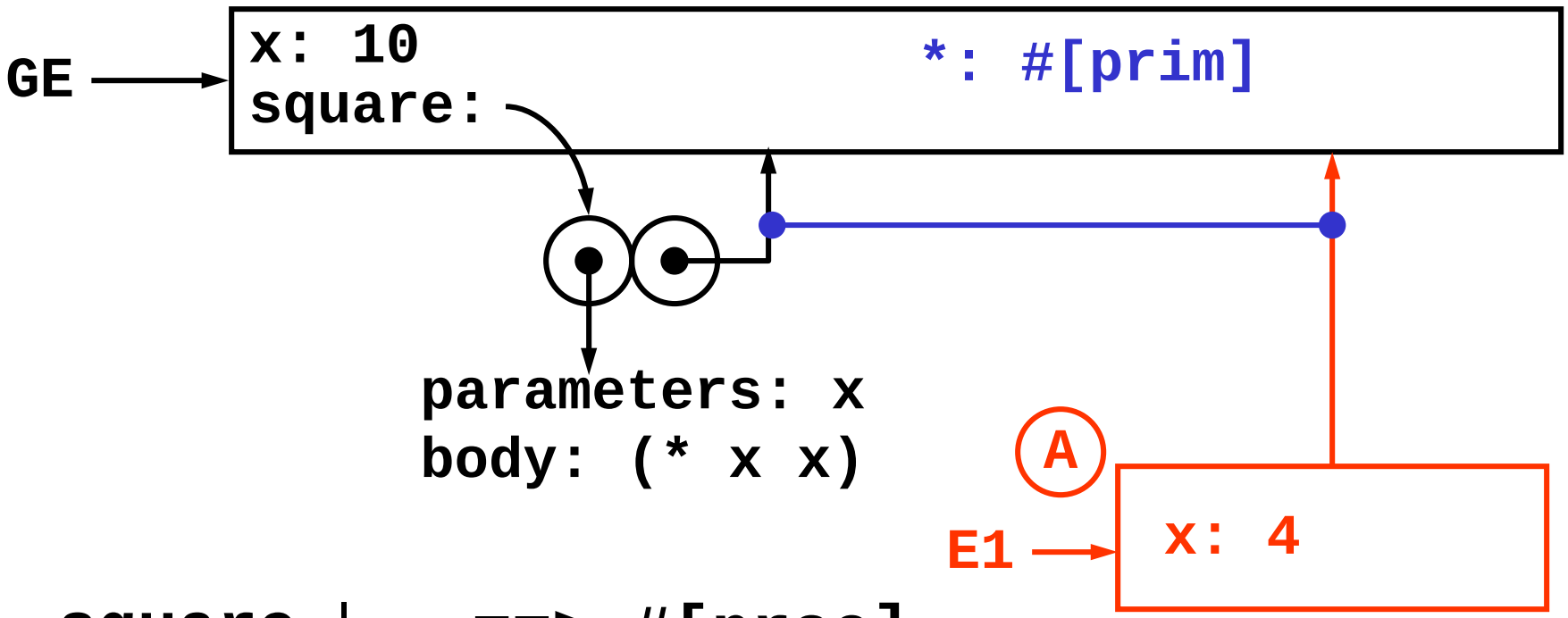
1. Create a new frame A
2. Make A into an environment E:
   A's enclosing environment pointer goes to *the same frame as the environment pointer of P*
3. In A, bind the parameters of P to the argument values
4. Evaluate the body of P with E as the current environment

Achieving Inner Peace (and A Good Grade), Part II



Om Mani Padme Hum…

(square 4) |GE



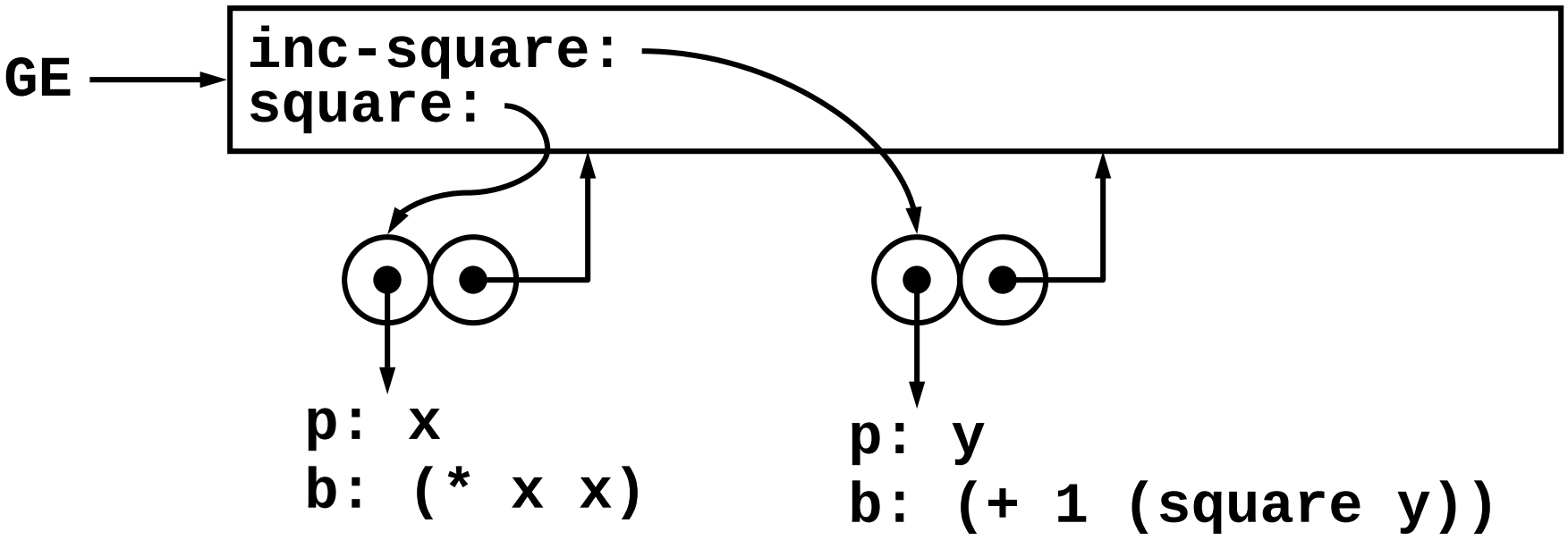
square |GE ==> #[proc]

(* x x) |E1 ==> 16

* |E1 ==> #[prim]

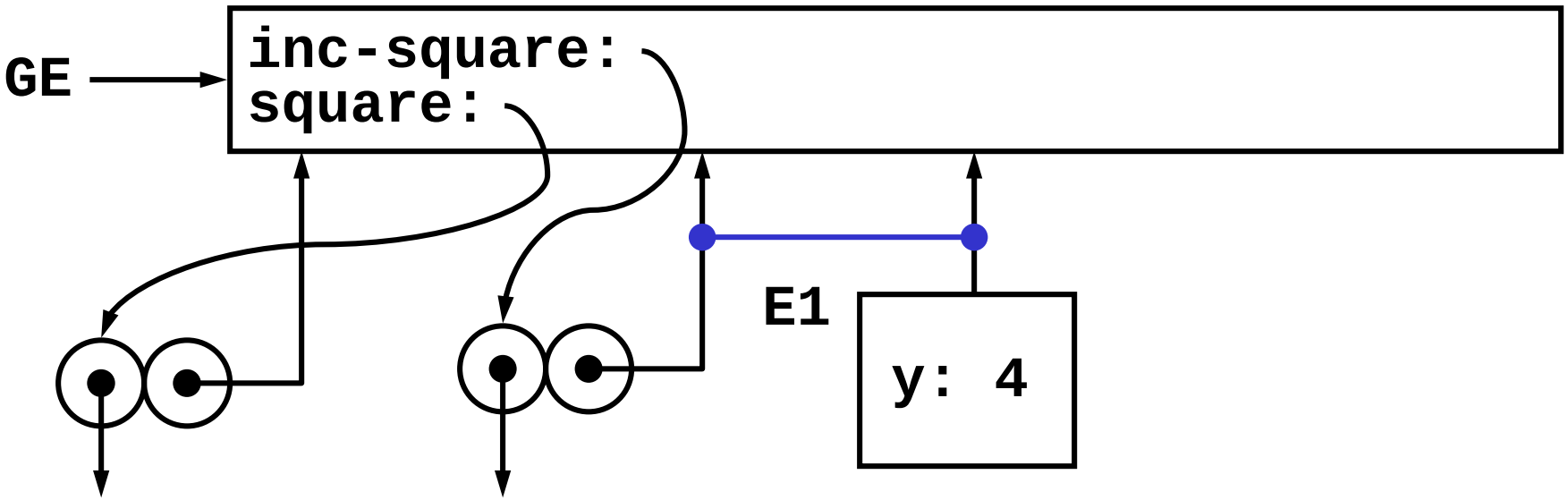
x |E1 ==> 4

Example: inc-square



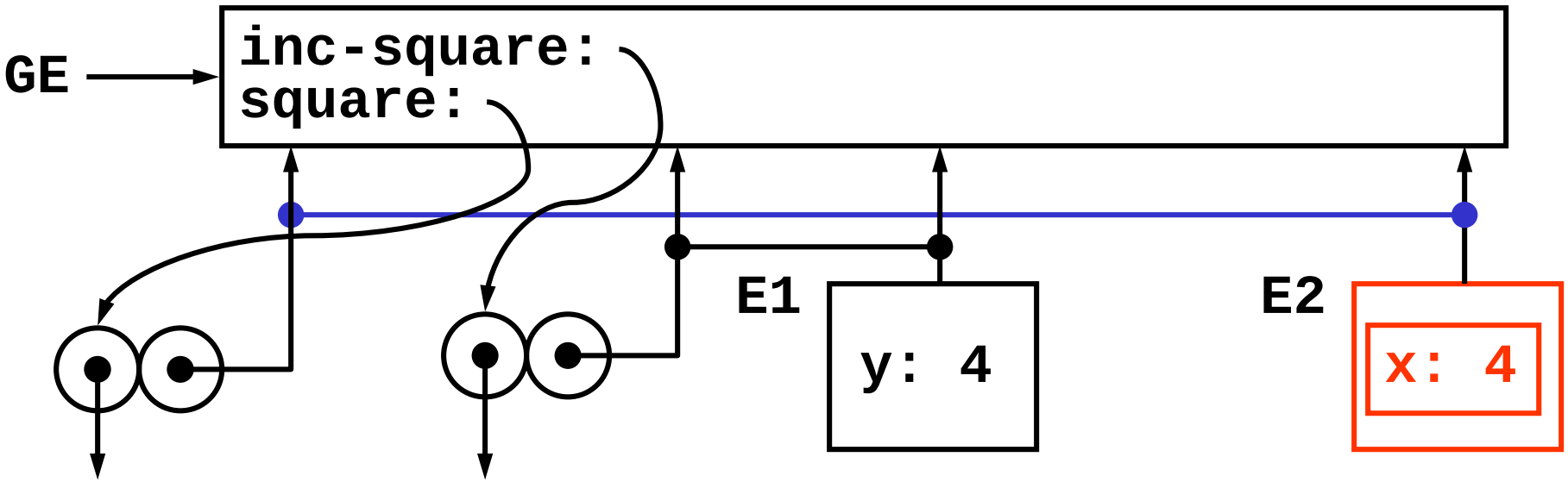

```
(define square (lambda (x) (* x x))) |GE
(define inc-square
          (lambda (y) (+ 1 (square y))) |GE
```

Example cont'd: (inc-square 4) |$_{GE}$



inc-square |$_{GE}$ ==> #[compound-proc ...]

Example cont'd: (square y) |$_{E1}$



square |$_{E1}$ ==> #[compound] y |$_{E1}$ ==> 4

(* x x) |$_{E2}$ ==> 16 (+ 1 16) ==> 17

* |$_{E2}$ ==> #[prim] x |$_{E2}$ ==> 4

Lessons from the `inc-square` example

- EM doesn't show the complete state of the interpreter
  - missing the stack of pending operations

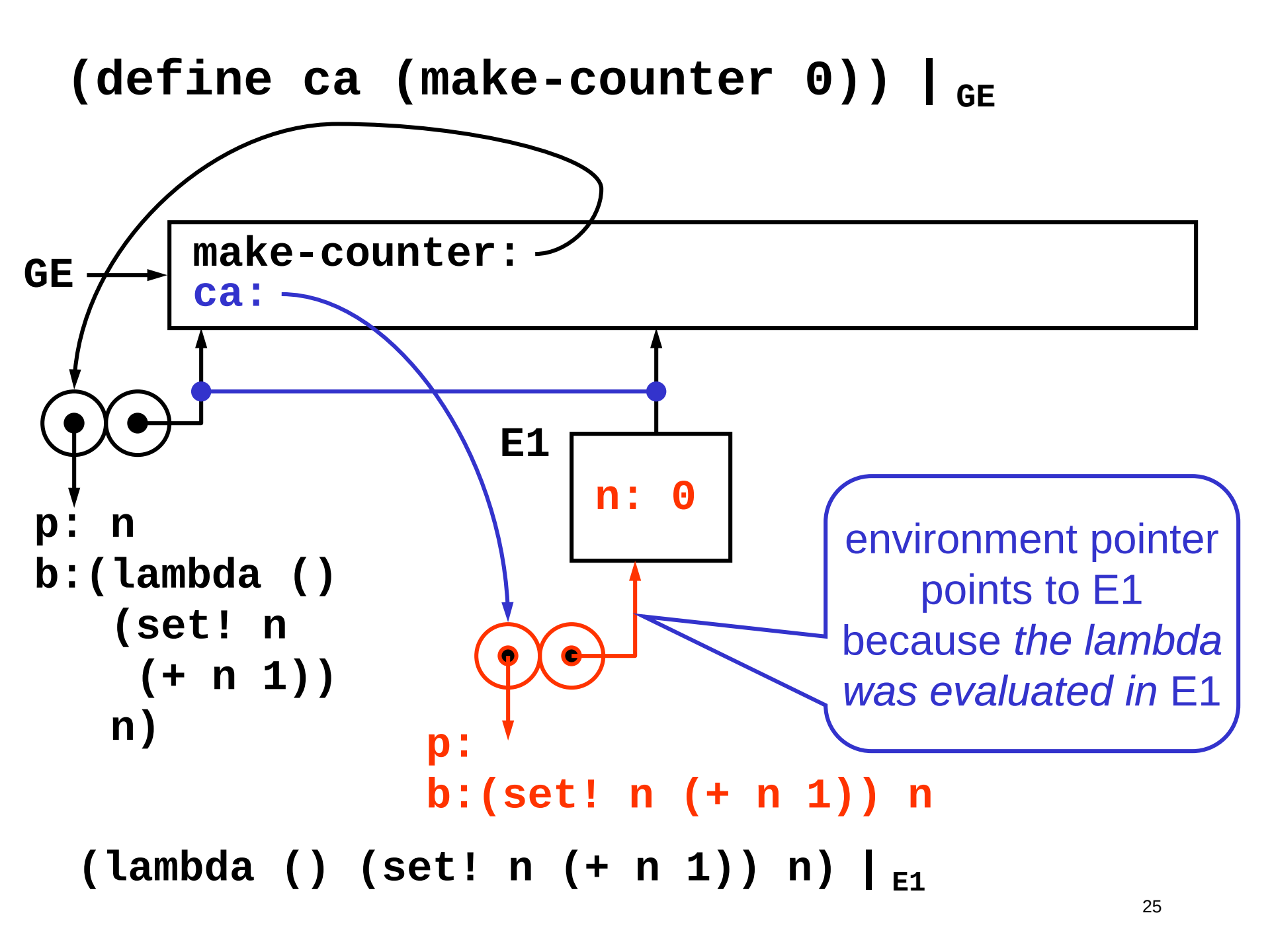
- The GE contains all standard bindings (**`*`**, **`cons`**, etc)
  - omitted from EM drawings

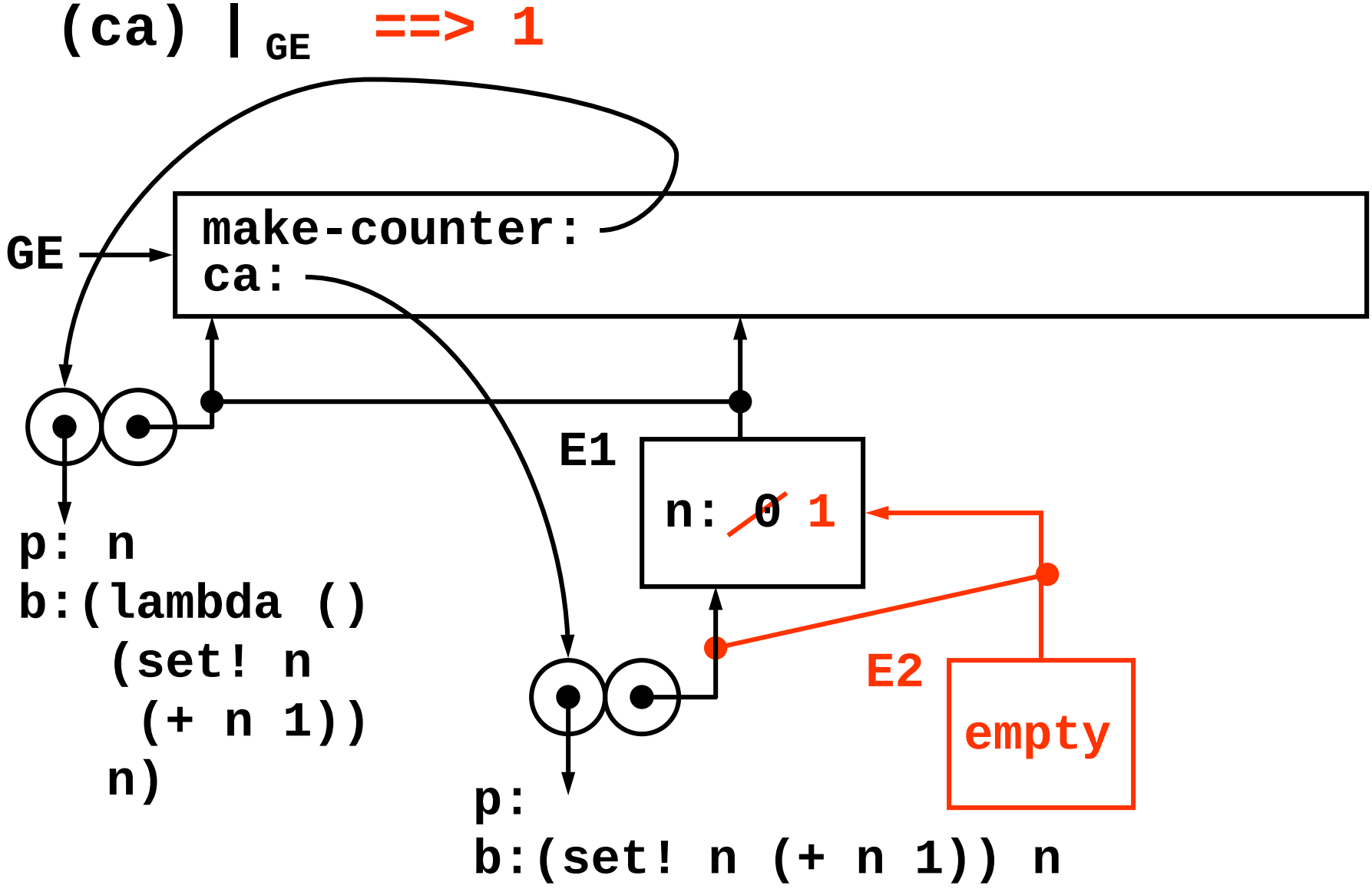
- Useful to link environment pointer of each frame to the procedure that created it

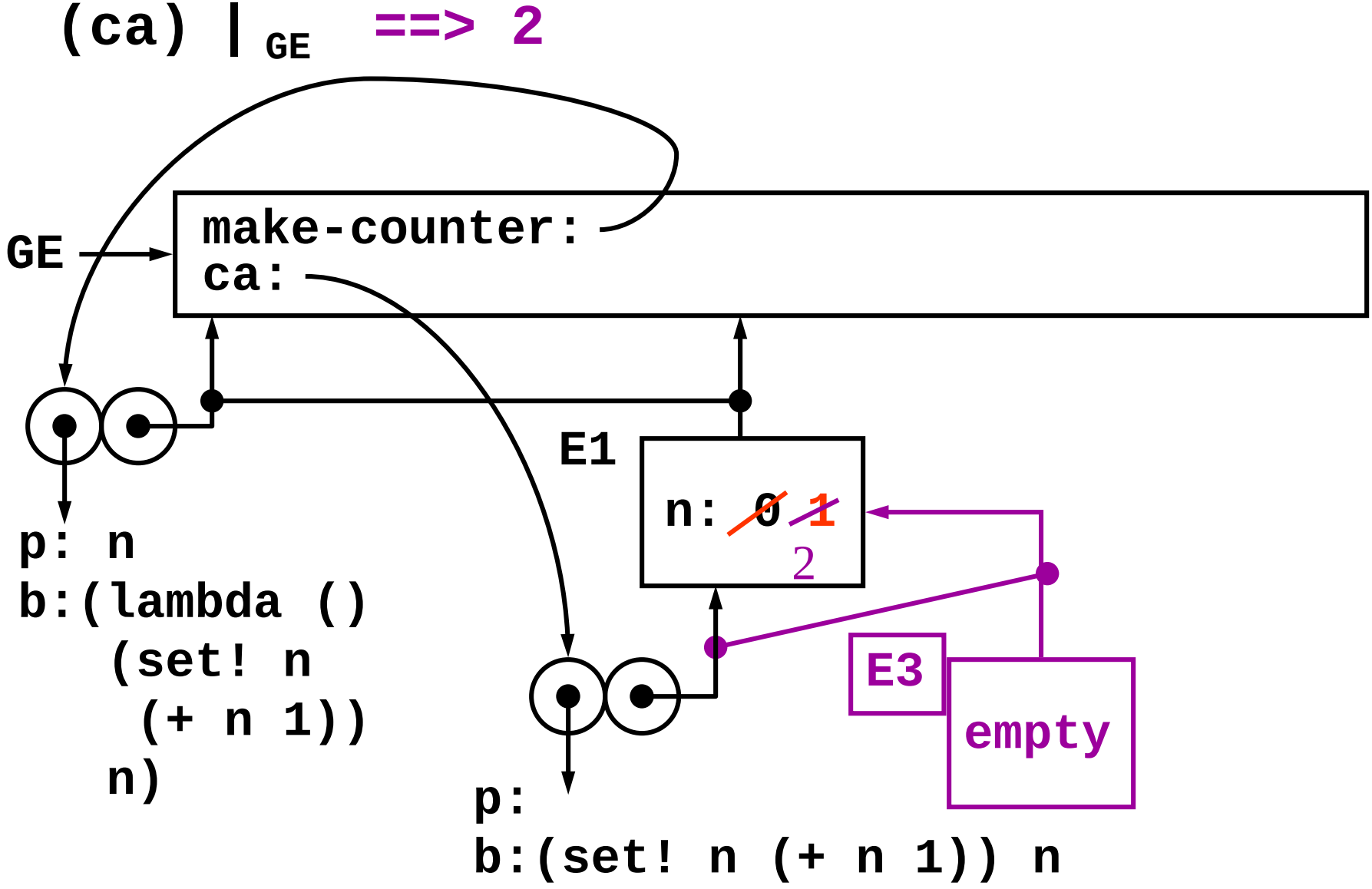
Example: make-counter

- Counter: something which counts up from a number

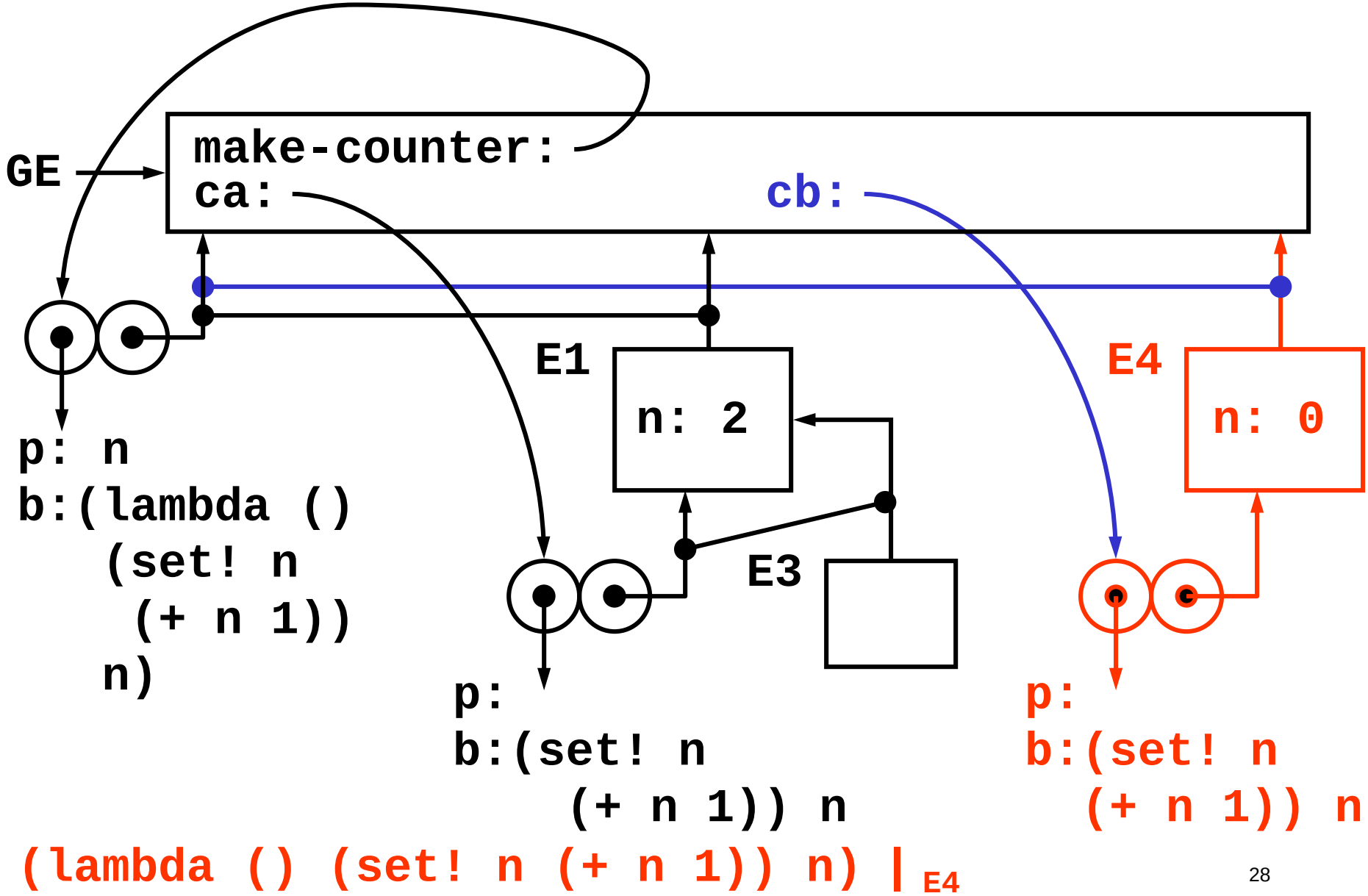
```
(define make-counter
  (lambda (n)
    (lambda () (set! n (+ n 1))
               n
    )))

(define ca (make-counter 0))
(ca) ==> 1
(ca) ==> 2 ; not functional programming
(define cb (make-counter 0))
(cb) ==> 1
(ca) ==> 3
(cb) ==> 2  ; ca and cb are independent
```


(define ca (make-counter 0)) | GE
GE
make-counter:
ca:
E1
n: 0
p: n
b:(lambda ()
(set! n
(+ n 1))
n)
p:
b:(set! n (+ n 1)) n
environment pointer points to E1 because *the lambda was evaluated in* E1
(lambda () (set! n (+ n 1)) n) | E1

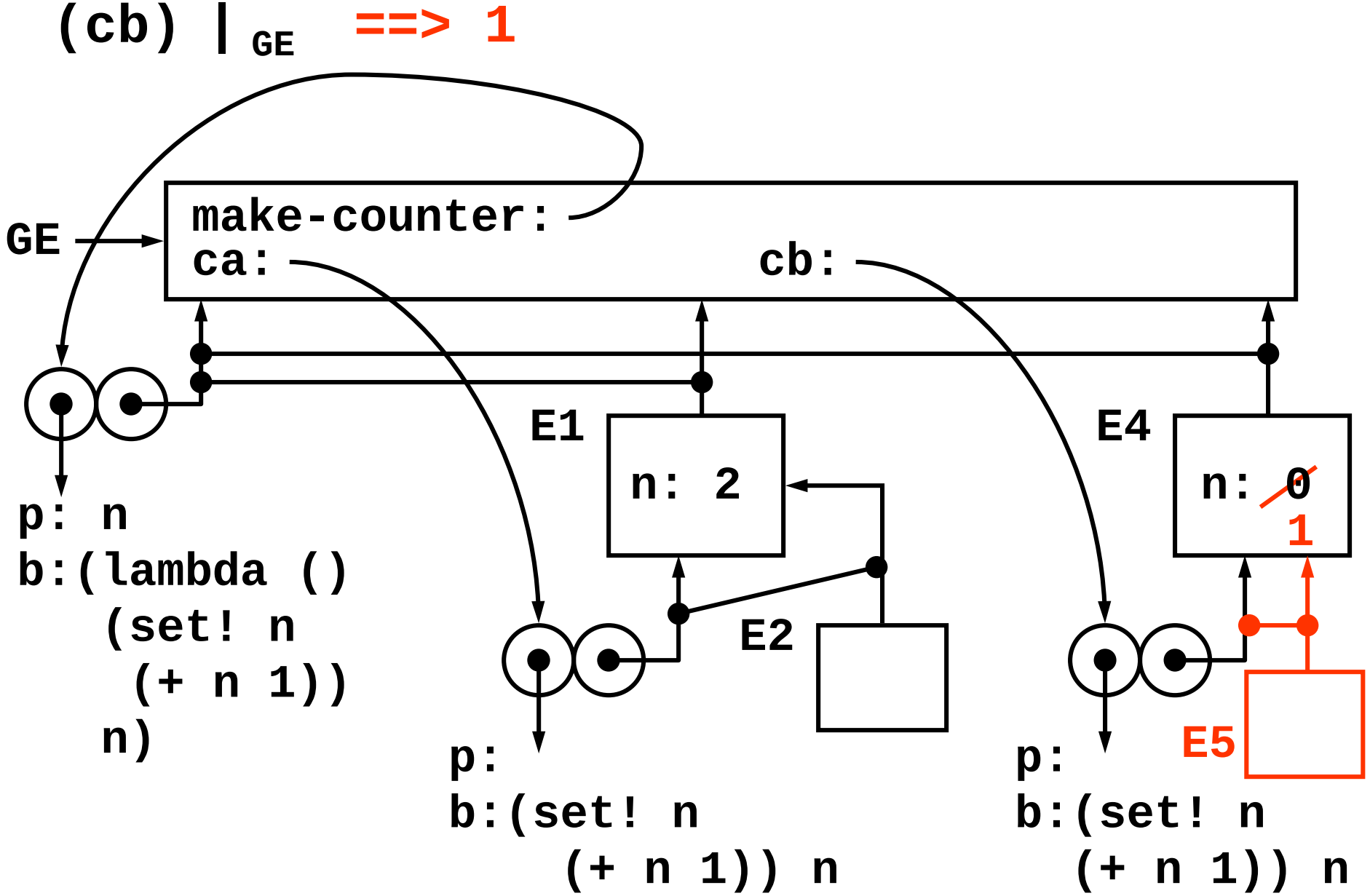

(ca) |GE ==> 1
GE
make-counter:
ca:
E1
n: 0 1
p: n
b:(lambda ()
(set! n
(+ n 1))
n)
E2
empty
p:
b:(set! n (+ n 1)) n
(set! n (+ n 1)) |E2
n |E2 ==> 1


(ca) |GE ==> 2
GE
make-counter:
ca:
E1
n: 0 1
2
p: n
b:(lambda ()
(set! n
(+ n 1))
n)
E3
empty
p:
b:(set! n (+ n 1)) n
(set! n (+ n 1)) |E3
n |E3 ==> 2

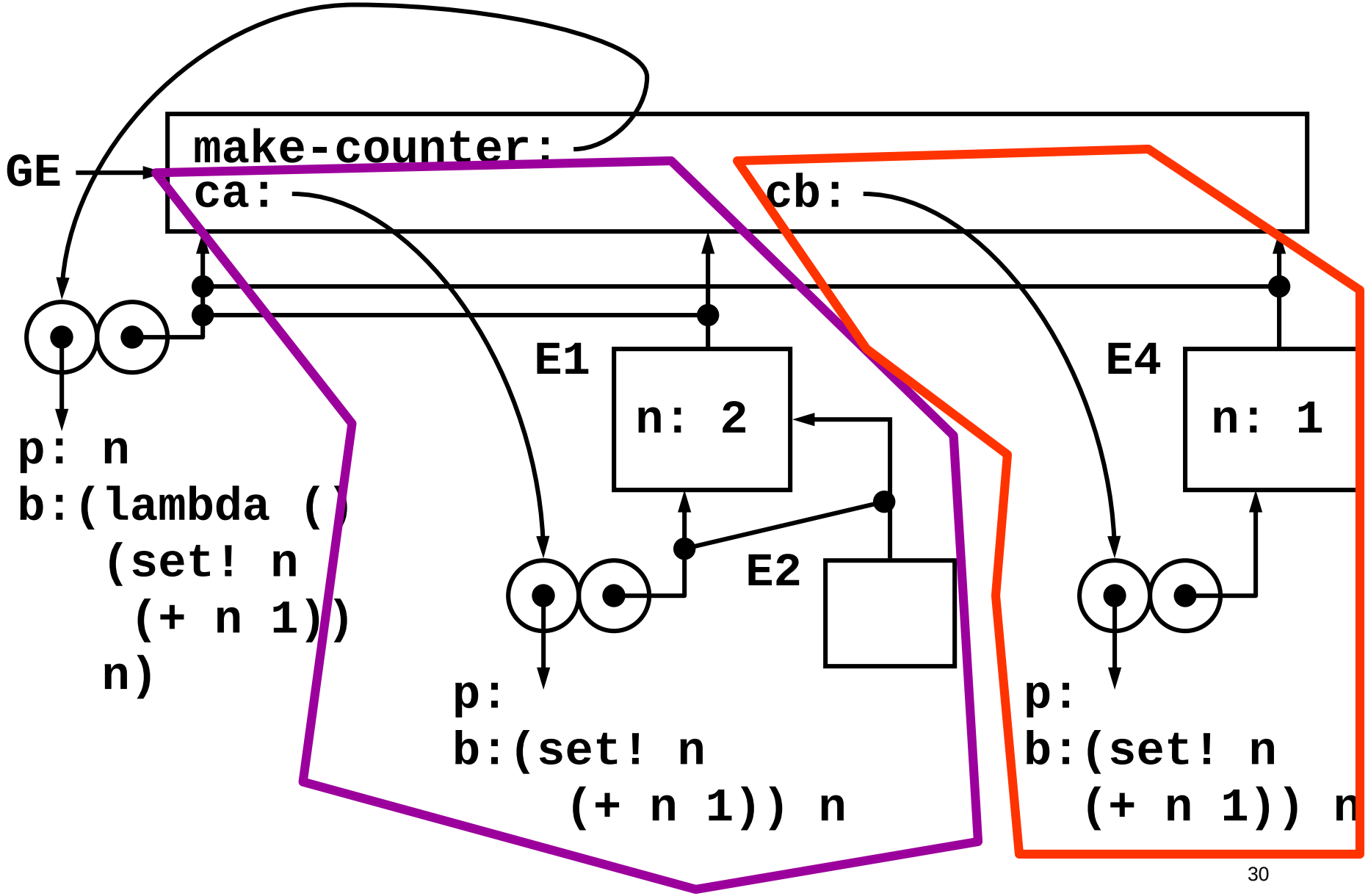
(define cb (make-counter 0)) | GE



(lambda () (set! n (+ n 1)) n) | E4

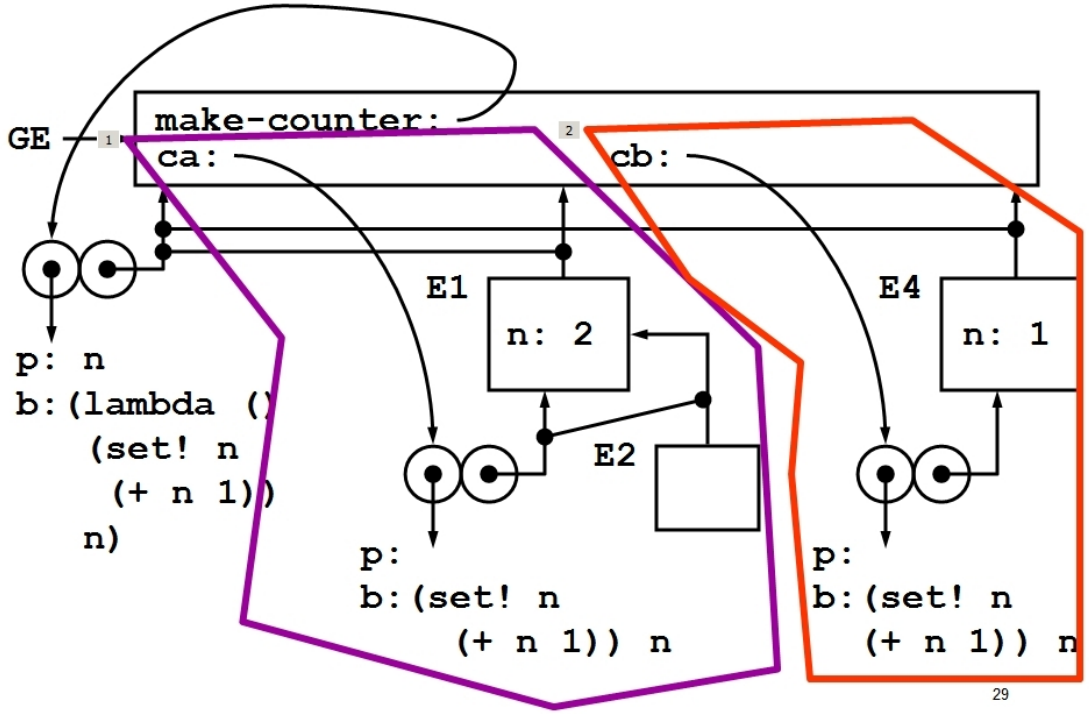

(cb) |GE ==> 1
GE
make-counter:
ca:
cb:
E1
n: 2
E4
n: 0
1
E2
E5
p: n
b:(lambda ()
(set! n
(+ n 1))
n)
p:
b:(set! n
(+ n 1)) n
p:
b:(set! n
(+ n 1)) n

Capturing state in local frames & procedures

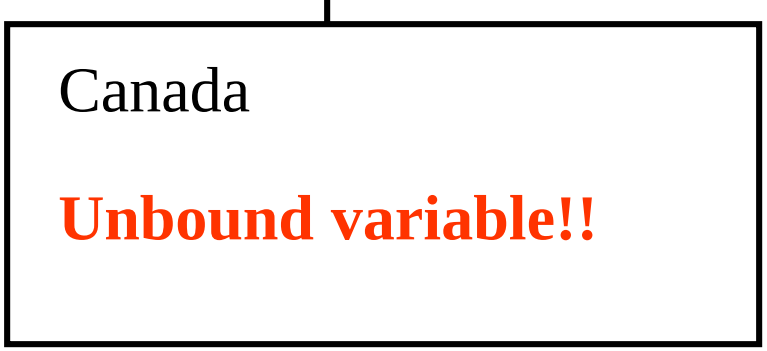


Lessons from the `make-counter` example

- Environment diagrams get complicated very quickly
  - Rules are meant for the computer to follow, not to help humans
- A lambda inside a procedure body captures the frame that was active when the lambda was evaluated
  - this effect can be used to store local state



Environments are important in other languages



Macintosh | USA

Milkshake | USA

Canadian bacon | New England

Macintosh | Britain

Milkshake | New England

Canadian bacon | Canada

Frappe | New England