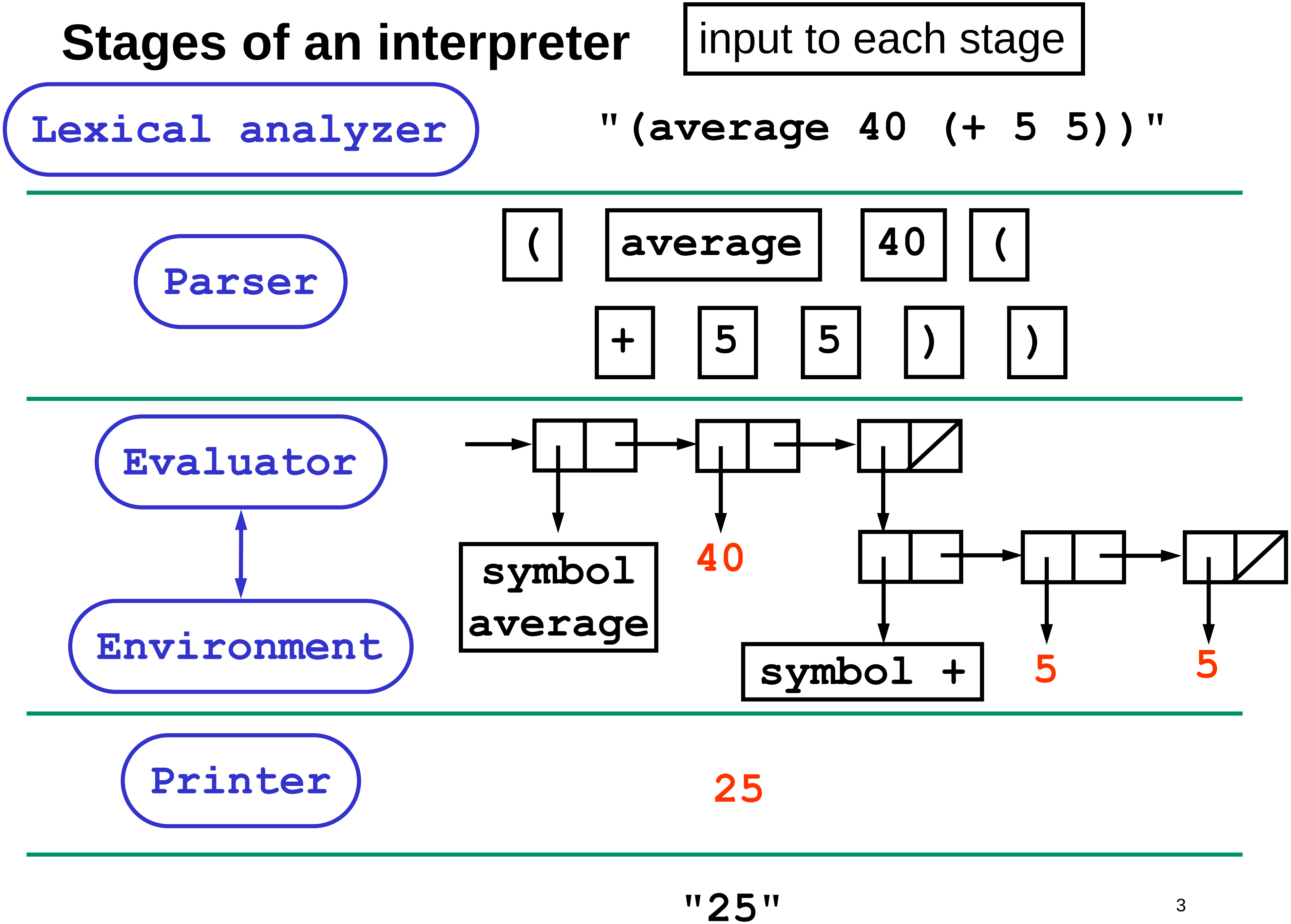
6.001 SICP
Interpretation

- Parts of an interpreter
- Arithmetic calculator
- Names
- Conditionals and if
- Storing procedures in the environment
- Environment as explicit parameter
- Defining new procedures

Why do we need an interpreter?

- Abstractions let us bury details and focus on use of modules to solve large systems
- We need a process to unwind abstractions at execution time to deduce meaning
- We have already seen such a process – the Environment Model
- Now want to describe that process as a procedure

Stages of an interpreter



Role of each part of the interpreter

- Lexical analyzer
  - break up input string into "words" called tokens
- Parser
  - convert linear sequence of tokens to a tree
  - like diagramming sentences in elementary school
  - also convert self-evaluating tokens to their internal values
    - e.g., **`#f`** is converted to the internal false value
- Evaluator
  - follow language rules to convert parse tree to a value
  - read and modify the environment as needed
- Printer
  - convert value to human-readable output string

Goal of today's lecture

- Implement an interpreter
- Only write evaluator and environment
  - Use Scheme's reader for lexical analysis and parsing
  - Use Scheme's printer for output
  - To do this, our language must resemble Scheme
- Call the language **`scheme*`**
  - All names end with a star to distinguish from Scheme names
- Start with interpreter for simple arithmetic expressions
  - Progressively add more features

1. Arithmetic calculator

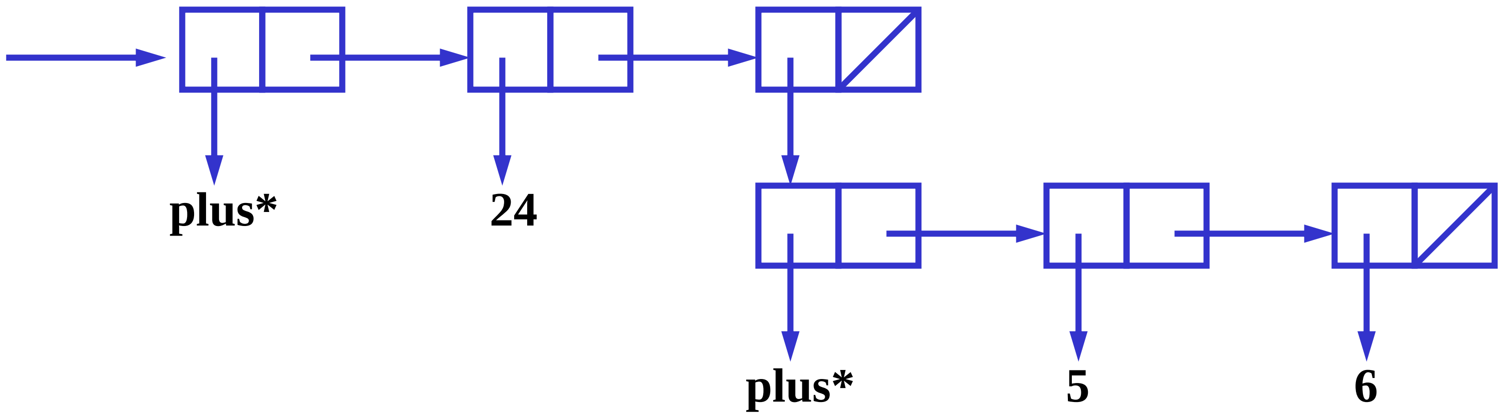
Want to evaluate arithmetic expressions of two arguments, like:

```
(plus* 24 (plus* 5 6))
```

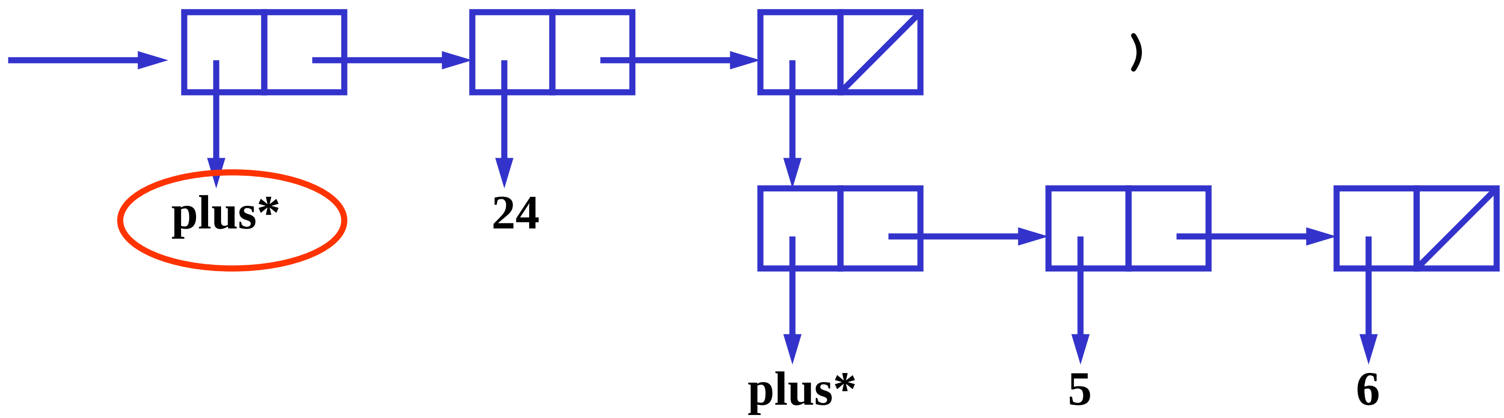
1. Arithmetic calculator

```
(define (tag-check e sym) (and (pair? e) (eq? (car e) sym)))
(define (sum? e) (tag-check e 'plus*))

(define (eval exp)
  (cond
   ((number? exp) exp)
   ((sum? exp)    (eval-sum exp))
   (else
    (error "unknown expression " exp))))

(define (eval-sum exp)
   (+ (eval (cadr exp)) (eval (caddr exp))))


(eval '(plus* 24 (plus* 5 6)))
```

We are just walking through a tree …



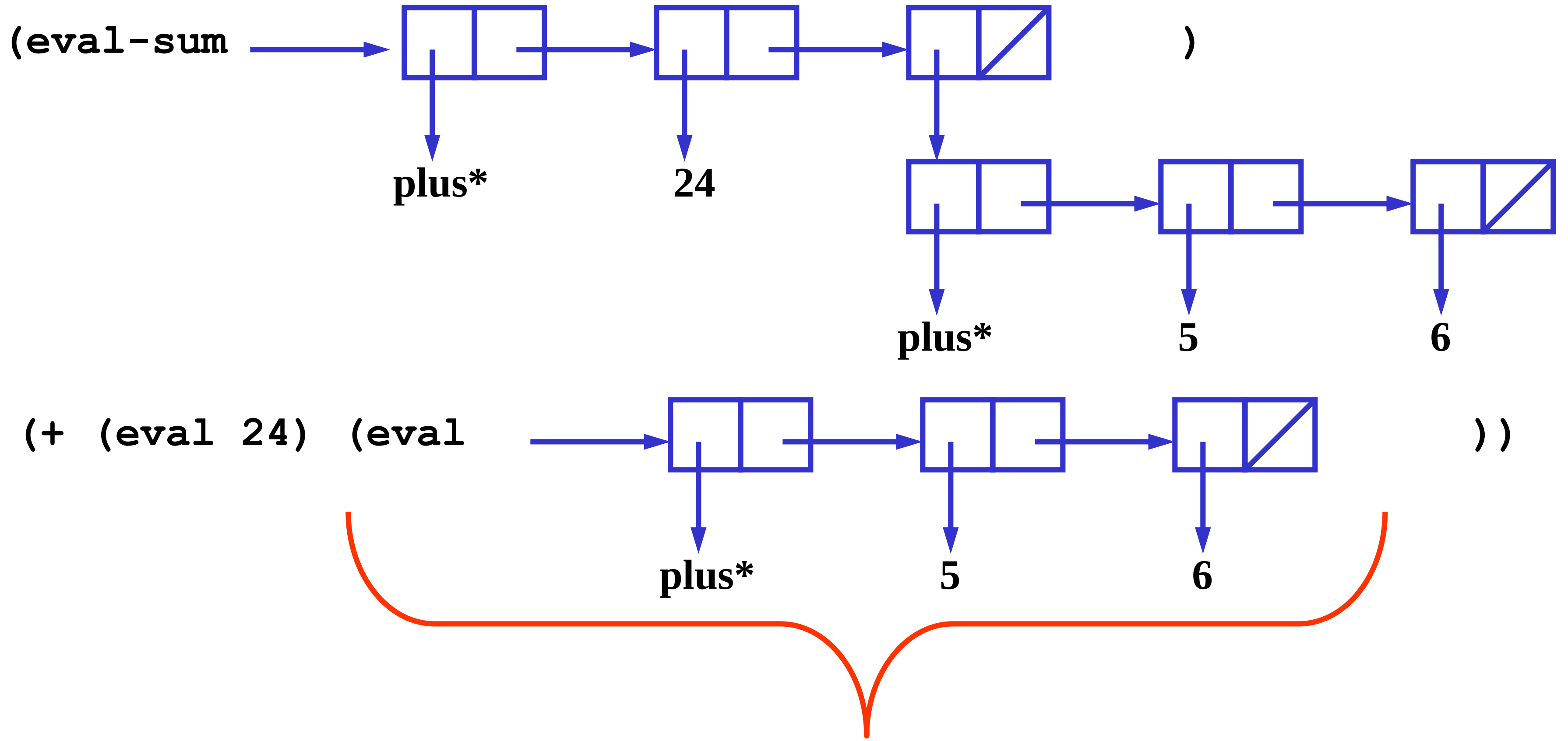
`(eval`



`)`

sum? checks the tag

We are just walking through a tree …

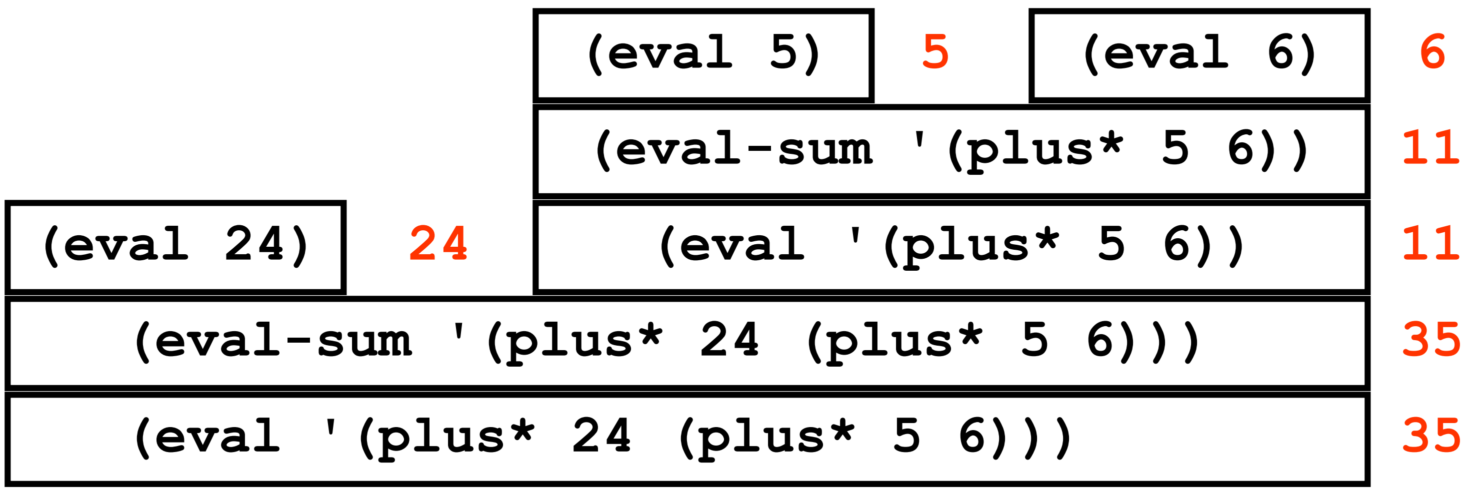



```
(+ (eval 5) (eval 6))
```

1. Arithmetic calculator

```
(plus* 24 (plus* 5 6))
```

- What are the argument and return values of **`eval`** each time it is called in the evaluation of this expression?



1. Things to observe

- **`cond`** determines the expression type
- No work to do on numbers
  - Scheme's reader has already done the work
  - It converts a sequence of characters like "24" to an internal binary representation of the number 24
- **`eval-sum`** recursively calls **`eval`** on both argument expressions

2. Names

- Extend the calculator to store intermediate results as named values

`(define* x* (plus* 4 5))`	store result as x*
`(plus* x* 2)`	use that result

- Store bindings between names and values in a table

2. Names

```
(define (define? exp) (tag-check exp 'define*))

(define (eval exp)
  (cond
   ((number? exp) exp)
   ((sum? exp)    (eval-sum exp))
   ((symbol? exp) (lookup exp))
   ((define? exp) (eval-define exp))
   (else
    (error "unknown expression " exp))))

; table ADT from prior lecture:
; make-table          void -> table
; table-get           table, symbol -> (binding | null)
; table-put!          table, symbol, anytype -> undef
; binding-value       binding -> anytype

(define environment (make-table))
```

2. Names ...

```
(define (lookup name)
  (let ((binding (table-get environment name)))
    (if (null? binding)
        (error "unbound variable: " name)
        (binding-value binding))))

(define (eval-define exp)
  (let ((name          (cadr exp))
        (defined-to-be (caddr exp)))
    (table-put! environment name (eval defined-to-be))
    'undefined))

(eval '(define* x* (plus* 4 5)))
(eval '(plus* x* 2))
```

How many times is **`eval`** called in these two evaluations?

Evaluation of page 2 lines 36 and 37

- Show argument and return values of **eval** for each call
- Show the environment each time it changes

```
(eval '(define* x* (plus* 4 5)))
     (eval '(plus* 4 5))
          (eval 4) ==> 4
          (eval 5) ==> 5
     ==> 9
 ==> undefined

 (eval '(plus* x* 2))
     (eval 'x*) ==> 9
     (eval 2) ==> 2
 ==> 11
```

environment

names	values
x*	9

2. Things to observe

- Use scheme function **`symbol?`** to check for a name
  - the reader converts sequences of characters like **`"x*"`** to symbols in the parse tree
- Can use any implementation of the **`table`** ADT
- **`eval-define`** recursively calls **`eval`** on the second subtree but not on the first one
- **`eval-define`** returns a special undefined value

3. Conditionals and if

- Extend the calculator to handle predicates and **`if`**:

```
(if* (greater* y* 6) (plus* y* 2) 15)
```

`greater*`	an operation that returns a boolean
`if*`	an operation that evaluates the first subexp, and checks if its value is true or false

- What are the argument and return values of **`eval`** each time it is called in the expression above?

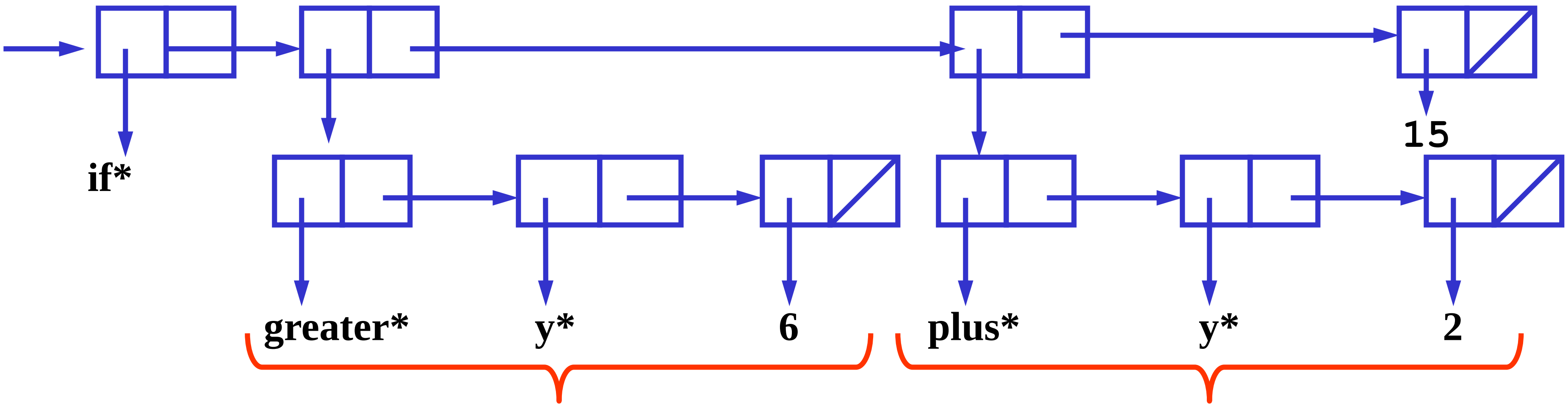
```
(define (greater? exp) (tag-check exp 'greater*))
(define (if? exp)      (tag-check exp 'if*))
```

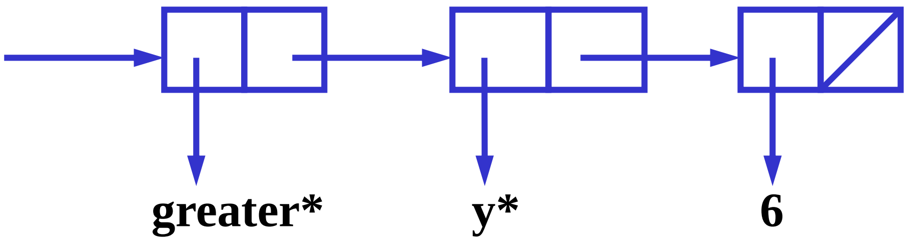
3. Conditionals and If

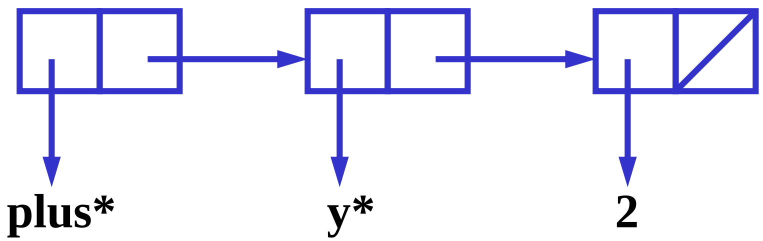
```
(define (eval exp)
  (cond ...
   ((greater? exp) (eval-greater exp))
   ((if? exp)      (eval-if exp))
   (else (error "unknown expression " exp))))

(define (eval-greater exp)
  (> (eval (cadr exp)) (eval (caddr exp))))

(define (eval-if exp)
  (let ((predicate   (cadr exp))
        (consequent  (caddr exp))
        (alternative (cadddr exp)))
    (let ((test (eval predicate)))
      (cond
       ((eq? test #t)  (eval consequent))
       ((eq? test #f)  (eval alternative))
       (else           (error "predicate not boolean: "
                              predicate))))))
(eval '(define* y* 9))
(eval '(if* (greater* y* 6) (plus* y* 2) 15))
```

Note: if* is stricter than Scheme's if

We are just walking through a tree …



`(eval`  `)`

Then `(eval`  `)` or `(eval 15 )`

Evaluation of page 3 line 32

```
(eval '(if* (greater* y* 6) (plus* y* 2) 15))
   (eval '(greater* y* 6))
       (eval 'y*) ==> 9
       (eval 6) ==> 6
   ==> #t
   (eval '(plus* y* 2))
       (eval 'y*) ==> 9
       (eval 2) ==> 2
   ==> 11
==> 11
```

3. Things to observe

- `eval-greater` is just like `eval-sum` from page 1
  - recursively call `eval` on both argument expressions
  - call Scheme `>` to compute value

- `eval-if` does not call `eval` on all argument expressions:
  - call `eval` on the predicate
  - call `eval` either on the consequent or on the alternative **but not both**
  - this is the mechanism that makes `if*` ________.________

4. Store operators in the environment

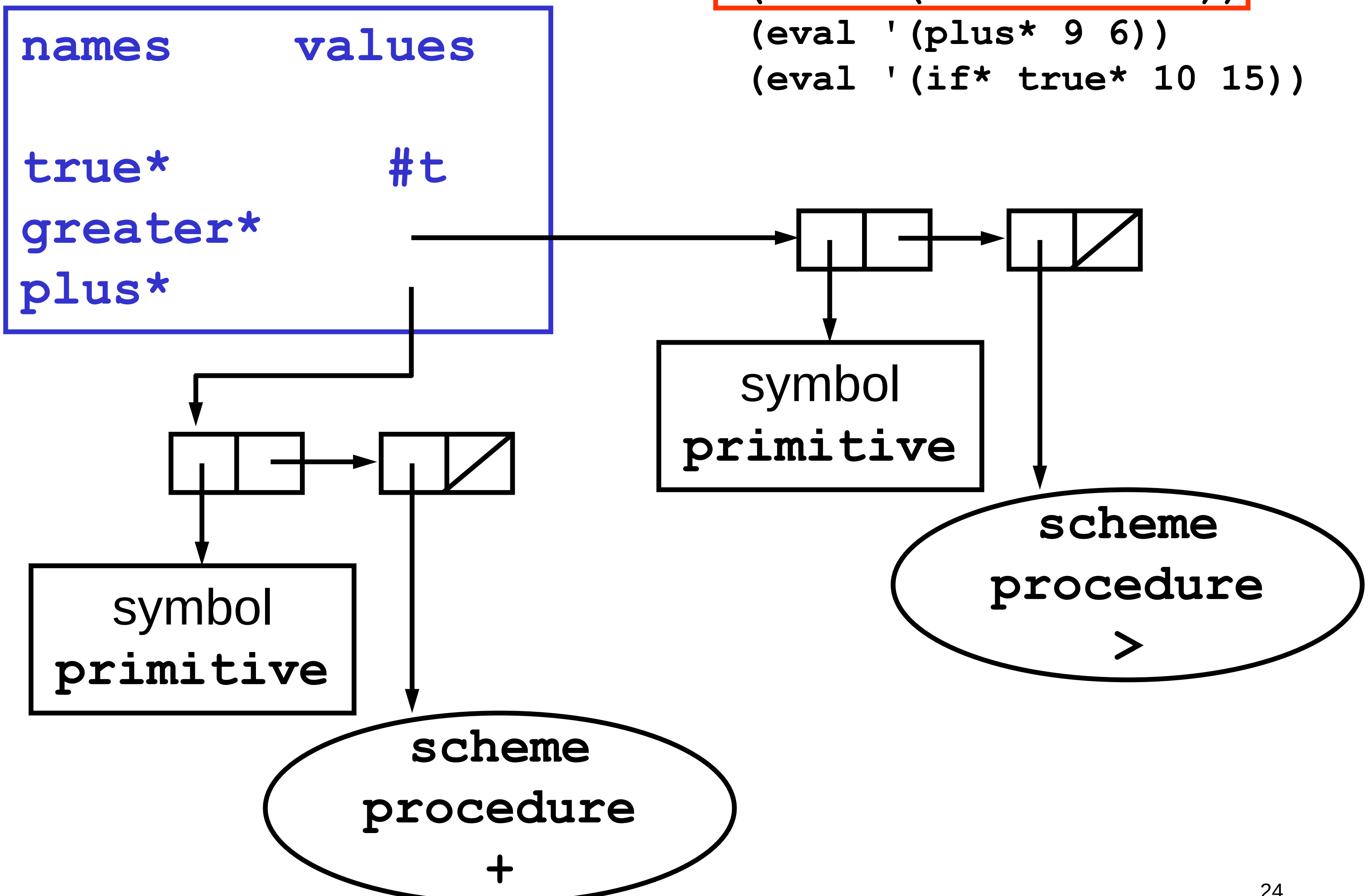
- Want to add lots of operators but keep **`eval`** short
- Operations like **`plus*`** and **`greater*`** are similar
  - evaluate all the argument subexpressions
  - perform the operation on the resulting values
- Call this standard pattern an application
  - Implement a single case in **`eval`** for all applications
- Approach:
  - **`eval`** the first subexpression of an application
  - put a name in the environment for each operation
  - value of that name is a procedure
  - apply the procedure to the operands

4. Store operators in the environment

```
(define (application? e) (pair? e))

(define (eval exp)
  (cond
   ((number? exp)      exp)
   ((symbol? exp)      (lookup exp))
   ((define? exp)      (eval-define exp))
   ((if? exp)          (eval-if exp))
   ((application? exp) (apply (eval (car exp))
                              (map eval (cdr exp))))
   (else
    (error "unknown expression " exp))))

(define scheme-apply apply) ;; rename scheme's apply so we can reuse the name

(define (apply operator operands)
  (if (primitive? operator)
      (scheme-apply (get-scheme-procedure operator) operands)
      (error "operator not a procedure: " operator)))

;; primitive: an ADT that stores scheme procedures

(define prim-tag 'primitive)
(define (make-primitive scheme-proc)(list prim-tag scheme-proc))
(define (primitive? e)              (tag-check e prim-tag))
(define (get-scheme-procedure prim) (cadr prim))

(define environment (make-table))
(table-put! environment 'plus*    (make-primitive +))
(table-put! environment 'greater* (make-primitive >))
(table-put! environment 'true* #t)
```

Environment after eval 4 line 36

```
(eval '(define* z* 9))
(eval '(plus* 9 6))
(eval '(if* true* 10 15))
```



Evaluation of eval 4 line 37

evaluating a combination…

```
(eval '(plus* 9 6))
(apply (eval 'plus*) (map eval '(9 6)))
(apply '(primitive #[add])
           (list (eval 9) (eval 6))
(apply '(primitive #[add]) '(9 6))
(scheme-apply
   (get-scheme-procedure '(primitive #[add]))
   '(9 6))
(scheme-apply #[add] '(9 6))
15
```

…turns into applying a proc to a set of values

Evaluation of eval 4 line 38

```
(eval '(if* true* 10 15))
(eval-if '(if* true* 10 15))
(let ((test (eval 'true*))) (cond ...))
(let ((test (lookup 'true*))) (cond ...))
(let ((test #t)) (cond ...))
(eval 10)
10
```

Apply is never called!

4. Things to observe

- applications must be the last case in **`eval`**
  - no tag check

- apply is never called in line 38
  - applications evaluate all subexpressions
  - expressions that need special handling, like **`if*`**, gets their own case in **`eval`**

5. Environment as explicit parameter

- Change from

```
(eval '(plus* 6 4))
```

  to

```
(eval '(plus* 6 4) environment)
```

- All procedures that call **`eval`** now have extra argument
- **`lookup`** and **`define`** use environment from argument

- No other change from evaluator 4

- Only nontrivial code: case for **`application?`** in eval

5. Environment as explicit parameter

```
(define (eval exp env)
  (cond
   ((number? exp)       exp)
   ((symbol? exp)       (lookup exp env))
   ((define? exp)       (eval-define exp env))
   ((if? exp)           (eval-if exp env))
   ((application? exp) (apply (eval (car exp) env)
                               (map (lambda (e) (eval e env))
                                    (cdr exp))))
   (else (error "unknown expression " exp))))

(define (lookup name env)
  (let ((binding (table-get env name)))
    (if (null? binding)
        (error "unbound variable: " name)
        (binding-value binding))))

(define (eval-define exp env)
  (let ((name (cadr exp))
        (defined-to-be (caddr exp)))
    (table-put! env name (eval defined-to-be env))
    'undefined))

(define (eval-if exp env)
  (let ((predicate   (cadr exp))
        (consequent  (caddr exp))
        (alternative (cadddr exp)))
    (let ((test (eval predicate env)))
      (cond
       ((eq? test #t)  (eval consequent env))
       ((eq? test #f)  (eval alternative env))
       (else           (error "predicate not boolean: "
                              predicate))))))
```

This change is boring! Exactly the same functionality as #4.

```
(eval '(define* z* (plus* 4 5))
      environment)
(eval '(if* (greater* z* 6) 10 15)
      environment)
```

6. Defining new procedures

- Want to add new procedures
- For example, a **`scheme*`** procedure:

```
(define* twice* (lambda* (x*) (plus* x* x*)))
(twice* 4)
```

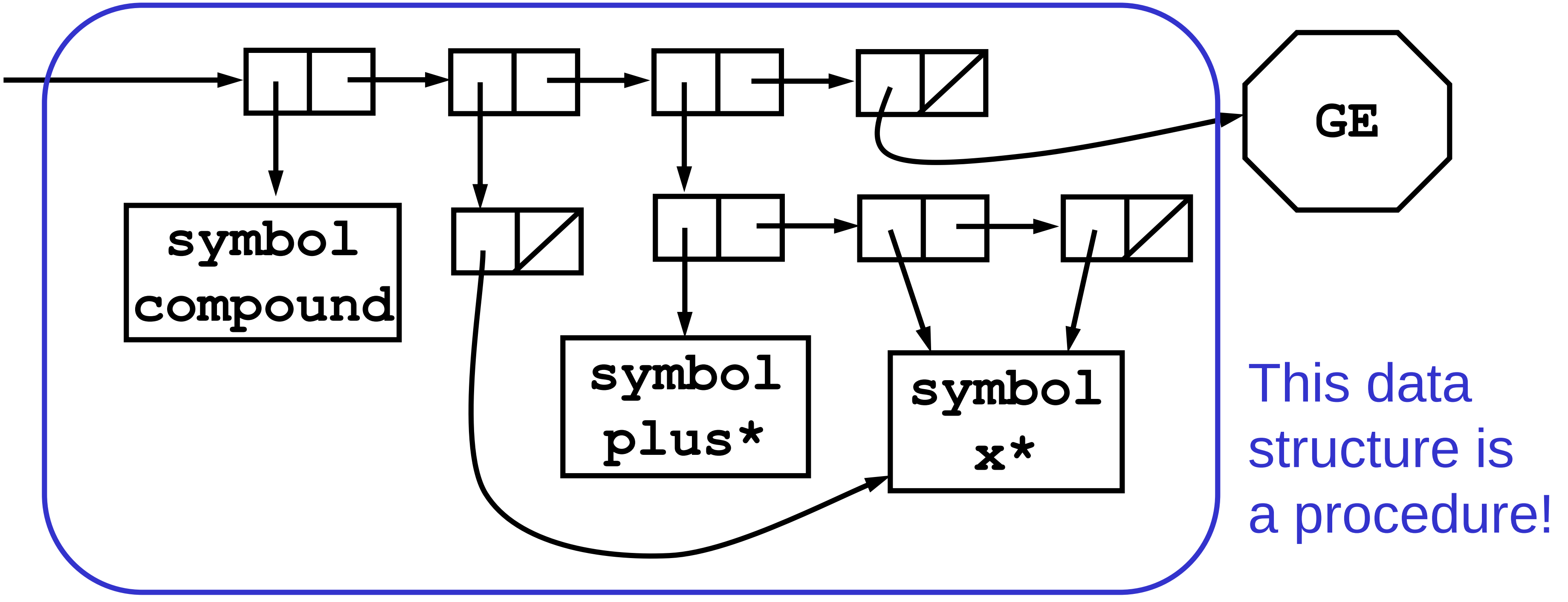
- Strategy:
  - Add a case for **`lambda*`** to **`eval`**
    - the value of **`lambda*`** is a compound procedure
  - Extend **`apply`** to handle compound procedures
  - Implement environment model

6. Defining new procedures

```
(define (lambda? e) (tag-check e 'lambda*))

(define (eval exp env)
  (cond  ((number? exp)       exp)
         ((symbol? exp)      (lookup exp env))
         ((define? exp)      (eval-define exp env))
         ((if? exp)          (eval-if exp env))
         ((lambda? exp)      (eval-lambda exp env))
         ((application? exp) (apply (eval (car exp) env)
                                    (map (lambda (e) (eval e env))
                                         (cdr exp))))
         (else (error "unknown expression " exp))))

(define (eval-lambda exp env)
  (let ((args (cadr exp))
        (body (caddr exp)))
    (make-compound args body env)))

(define (apply operator operands)
  (cond ((primitive? operator)
         (scheme-apply (get-scheme-procedure operator) operands))
        ((compound? operator)
         (eval (body operator)
               (extend-env-with-new-frame
                            (parameters operator)
                            operands
                            (env operator))))
        (else (error "operator not a procedure: " operator))))

;; ADT that implements the "double bubble"
(define compound-tag 'compound)
(define (make-compound parameters body env)
                    (list compound-tag parameters body env))
(define (compound? exp)  (tag-check exp compound-tag))
(define (parameters compound) (cadr compound))
(define (body compound)       (caddr compound))
(define (env compound)        (cadddr compound))
```

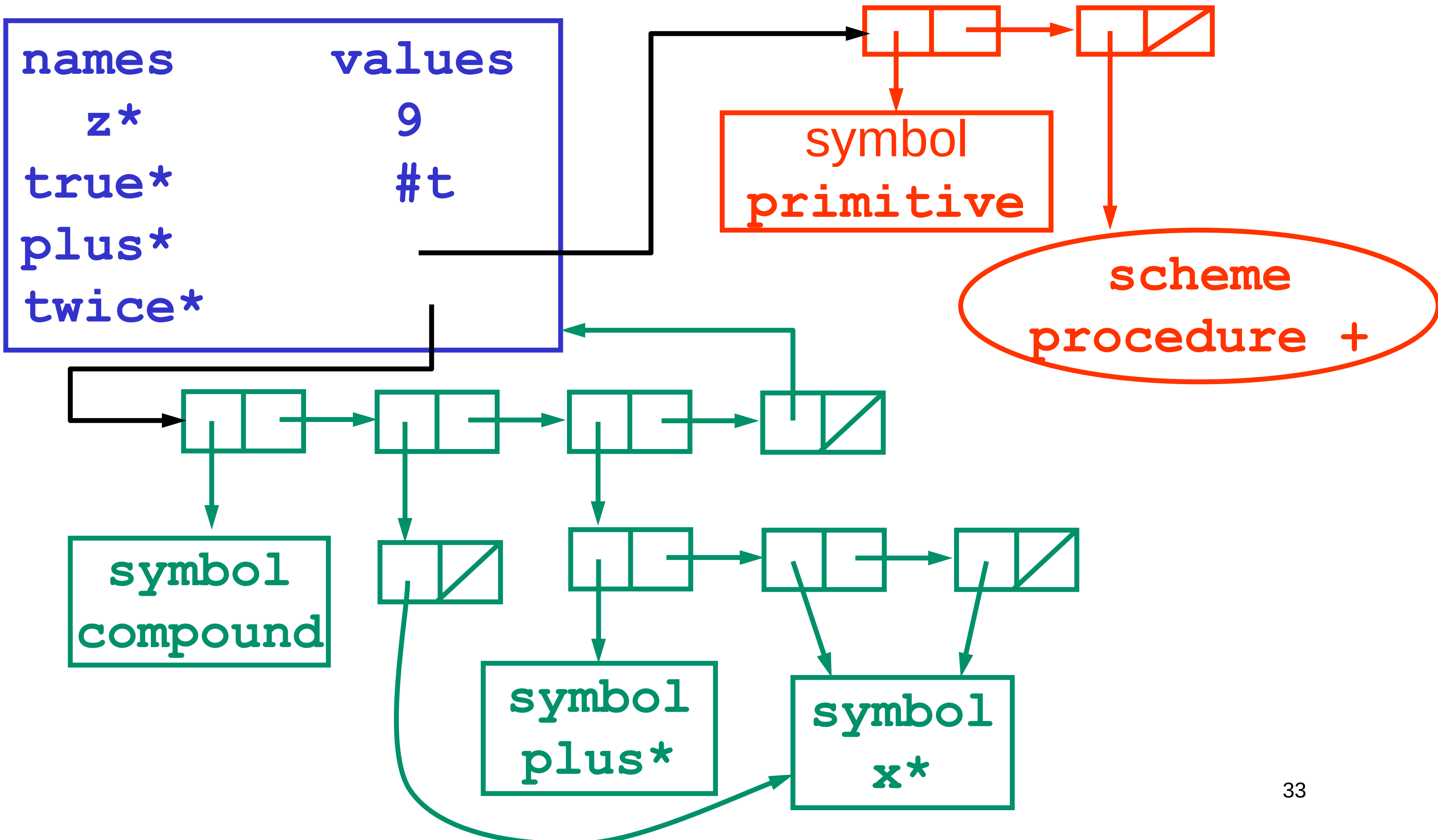
Implementation of `lambda*`

```
(eval '(lambda* (x*) (plus* x* x*)) GE)
(eval-lambda '(lambda* (x*) (plus* x* x*)) GE)
(make-compound '(x*) '(plus* x* x*) GE)
(list 'compound '(x*) '(plus* x* x*) GE)
```



Defining a named procedure

```
(eval '(define* twice*
            (lambda* (x*) (plus* x* x*))) GE)
```



Implementation of `apply` (1)

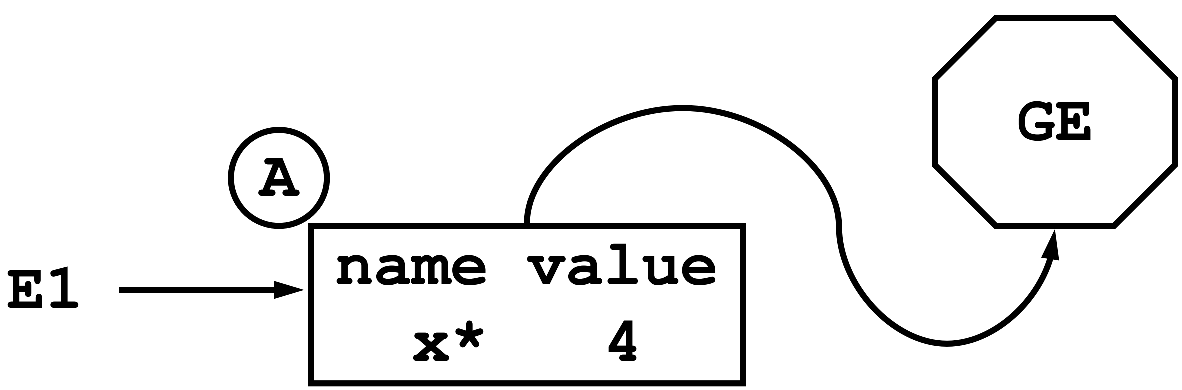
(eval '(twice* 4) ~~GE~~) some-other-environment)

(apply (eval 'twice* ~~GE~~)
       (map (lambda (e) (eval e ~~GE~~)) '(4)))

(apply (list 'compound '(x*) '(plus* x* x*) GE)
       '(4))

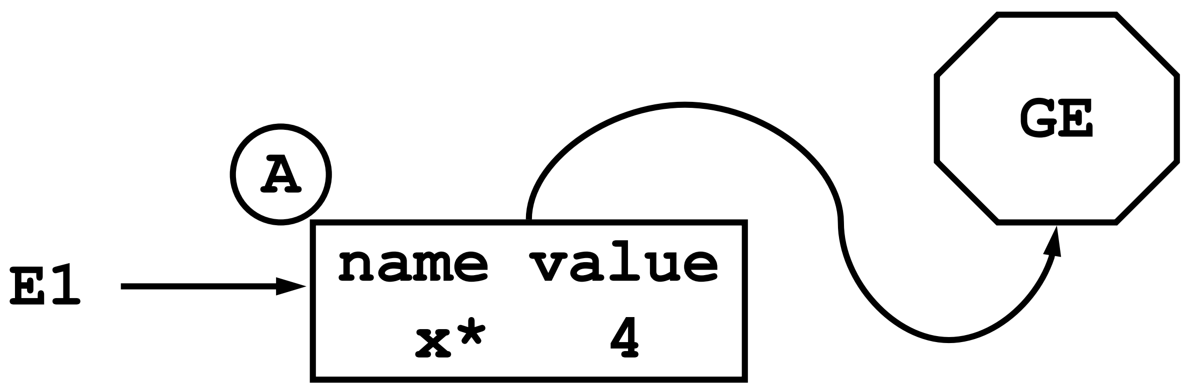
(eval '(plus* x* x*)
      (extend-env-with-new-frame '(x*) '(4) GE))

(eval '(plus* x* x*) E1)



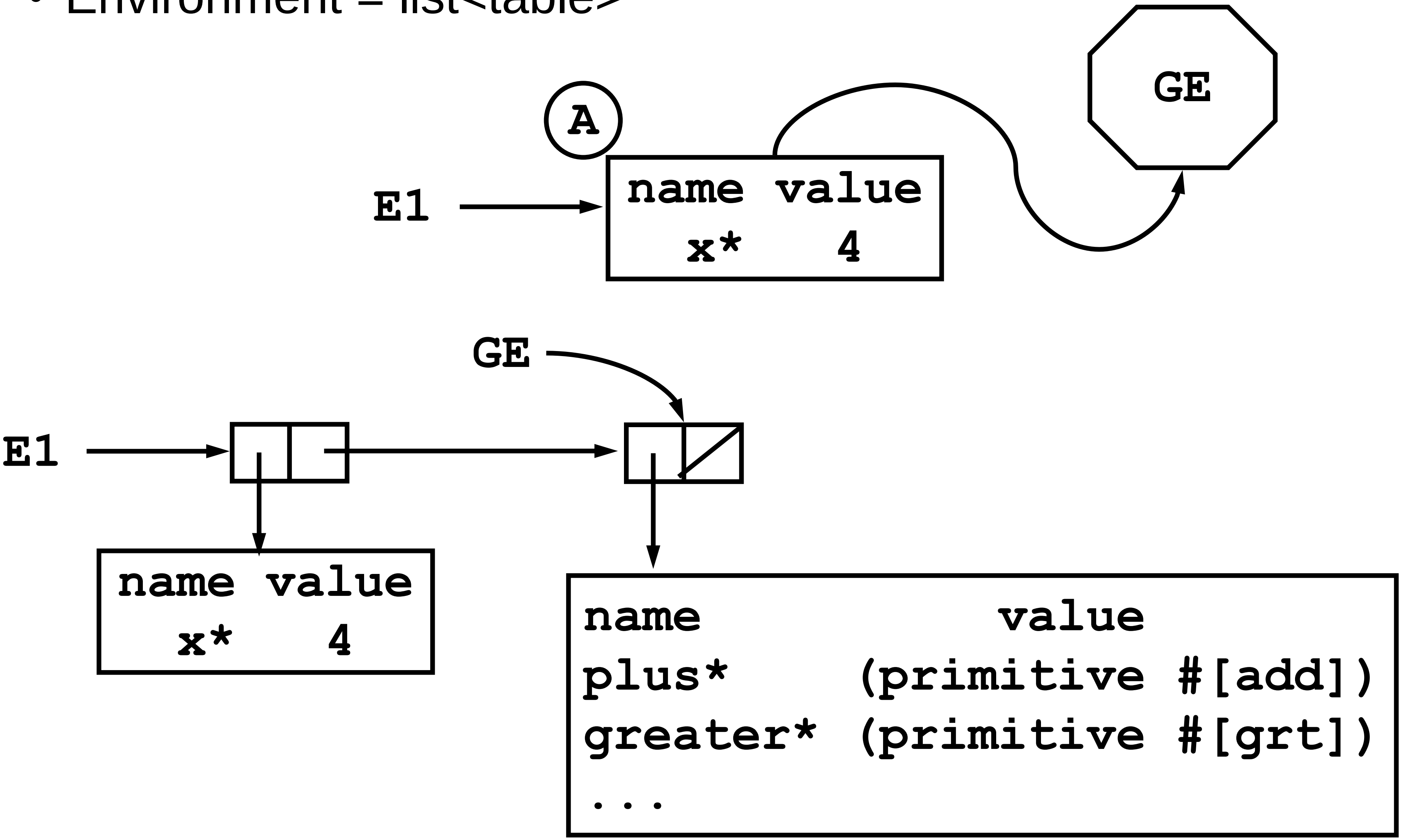
Implementation of `apply` (2)

```
(eval '(plus* x* x*) E1)
(apply (eval 'plus* E1)
       (map (lambda (e) (eval e E1)) '(x* x*)))
(apply '(primitive #[add]) (list (eval 'x* E1)
                                 (eval 'x* E1)))
(apply '(primitive #[add]) '(4 4))
(scheme-apply #[add] '(4 4))
8
```



Implementation of environment model

- Environment = list<table>



; Environment model code (part of eval 6)

```
; Environment = list<table>

(define (extend-env-with-new-frame names values env)
  (let ((new-frame (make-table)))
    (make-bindings! names values new-frame)
    (cons new-frame env)))

(define (make-bindings! names values table)
  (for-each
     (lambda (name value) (table-put! table name value))
    names values))

; the initial global environment
(define GE
  (extend-env-with-new-frame
     (list 'plus* 'greater*)
     (list (make-primitive +) (make-primitive >))
    nil))

; lookup searches the list of frames for the first match
(define (lookup name env)
  (if (null? env)
      (error "unbound variable: " name)
      (let ((binding (table-get (car env) name)))
        (if (null? binding)
            (lookup name (cdr env))
            (binding-value binding)))))

; define changes the first frame in the environment
(define (eval-define exp env)
  (let ((name          (cadr exp))
        (defined-to-be (caddr exp)))
    (table-put! (car env) name (eval defined-to-be env))
    'undefined))

(eval '(define* twice* (lambda* (x*) (plus* x* x*))) GE)
(eval '(twice* 4) GE)
```

Summary

- Cycle between eval and apply is the core of the evaluator
  - eval calls apply with operator and argument values
  - apply calls eval with expression and environment
  - no pending operations on either call
    - an iterative algorithm if the expression is iterative

- What is still missing from **`scheme*`** ?
  - ability to evaluate a sequence of expressions
  - data types other than numbers and booleans

Cute Punchline

- *Everything in these lectures would still work if you deleted the stars from the names*.
- We just wrote (most of) a Scheme interpreter in Scheme.
- Seriously nerdly, eh?
  - The language makes things explicit
    – e.g., procedures and procedure app in environment
  - More generally
    – Writing a precise definition for what the Scheme language means
    – Describing computation in a computer language forces precision and completeness
    – Sets the foundation for exploring variants of Scheme