6.037 Lecture 3

Mutation and
The Environment Model

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Previously, on 6.037....

- Basics of Scheme
- Substitution Model
- Recursion, plus iterative and recursive processes
- Procedural abstraction
- Abstract data types (cons cells and lists)
- Higher-order procedures
- Symbols and quotation
- Tagged Data

Data Mutation

- Syntax
  - **`set!`** for names
  - **`set-car!`**, **`set-cdr!`** for pairs
- Semantics
  - Simple case: one global environment
  - Complex case: many environments: environment model

Primitive Data

`(define x 10)` — creates a new binding for name; special form

`x` — returns value bound to name

- To Mutate:

`(set! x "foo")` — changes the binding for name; special form (value is undefined)

Assignment -- set!

- Substitution model -- *functional programming*:

```
(define x 10)
(+ x 5) ==> 15
...
(+ x 5) ==> 15
```

- expression has same value each time it's evaluated (in same scope as binding)

- With mutation:

```
(define x 10)
(+ x 5) ==> 15
...
(set! x 94)
...
(+ x 5) ==> 99
```

- expression "value" depends on when it is evaluated

Syntax: Expression Sequences

- With side-effects, sometimes you want to do some things and then return a value. Use the **begin** special form.
- ```
  (begin
        (set! x 2)
        (set! y 3)
        4) ; return value
  ```
- **lambda, let,** and **cond** accept sequences

```
(define frob
   (lambda ()
      (display "frob called") ; do this
      (set! x (+ x 1))        ; then this
      x))
```
```

Mutating Compound Data

- constructor:

  `(cons x y)` creates a new pair **p**

- selectors:

  `(car p)` returns car part of pair **p**

  `(cdr p)` returns cdr part of pair **p**

- mutators:

  `(set-car! p new-x)` changes car part of pair **p**

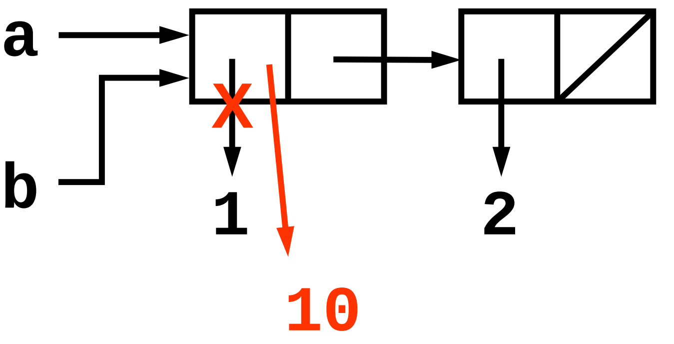
  `(set-cdr! p new-y)` changes cdr part of pair **p**

  `; Pair,anytype -> undef` **`--`** side-effect only!

Example 1: Pair/List Mutation

```
(define a (list 1 2))
(define b a)
a ➔ (1 2)
b ➔ (1 2)

(set-car! a 10)
b ➔ (10 2)
```

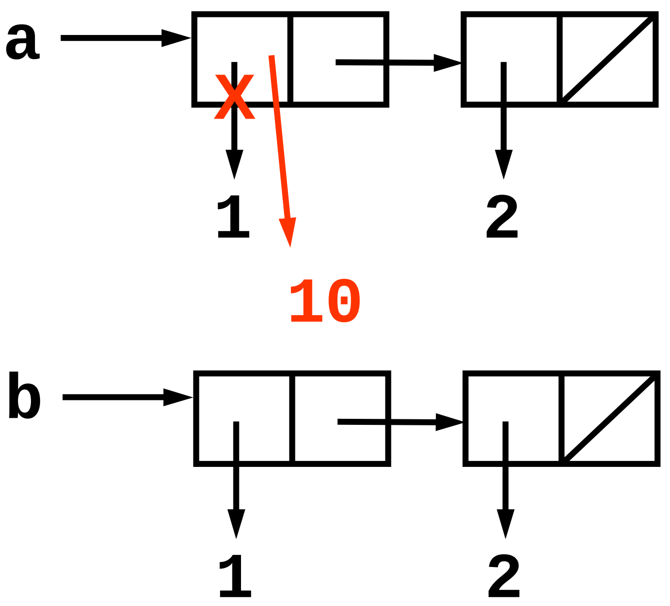


Compare with:

```
(define a (list 1 2))

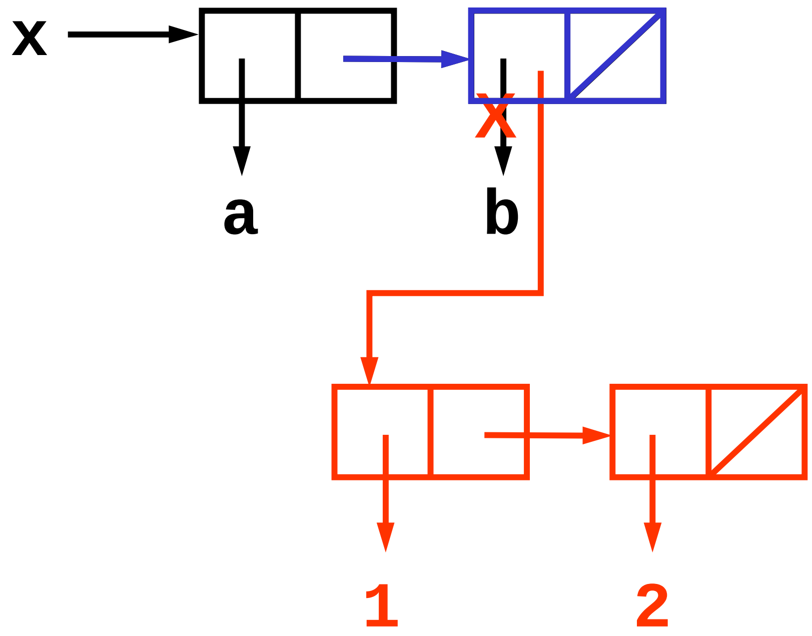
(define b (list 1 2))
(set-car! a 10)

b ➔ (1 2)
```



Example 2: Pair/List Mutation

```
(define x (list 'a 'b))
```



- How can we use mutation to achieve the result at right?

```
(set-car! (cdr x)
          (list 1 2))
```

1. Evaluate **`(cdr x)`** to get a pair object
2. Change car part of that pair object

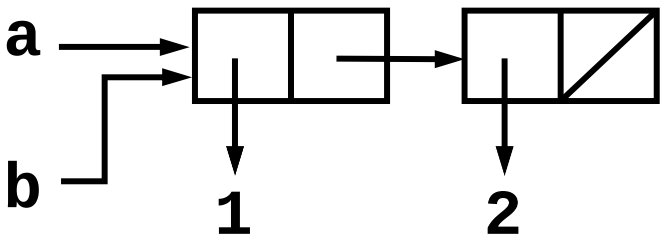
Sharing, Equivalence, and Identity

- How can we tell if two things are equivalent?

  Well, what do you mean by "equivalent"?

  - The *same object*: test with **`eq?`**

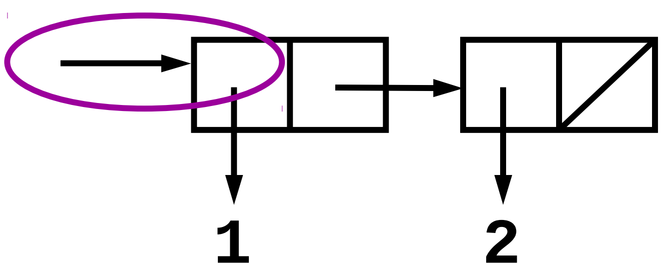
    `(eq? a b) ==> #t`

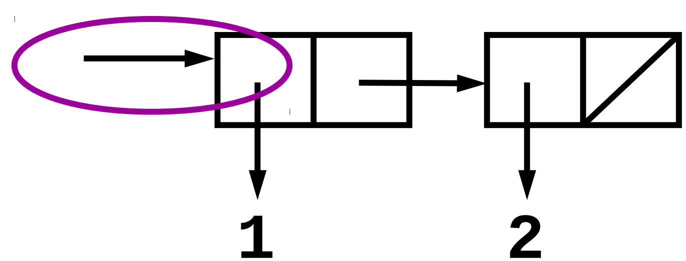


  - Objects that *"look" the same*: test with **`equal?`**

    `(equal? (list 1 2) (list 1 2)) ==> #t`

    `(eq?   (list 1 2) (list 1 2)) ==> #f`





`(1 2)` **`(1 2)`**

Sharing, Equivalence, and Identity

- How can we tell if two things are equivalent?

  Well, what do you mean by "equivalent"?

  - The *same object*: test with **`eq?`**

    `(eq? a b) ==> #t`

  - Objects that *"look" the same*: test with **`equal?`**

    ```
    (equal? (list 1 2) (list 1 2)) ==> #t
    (eq?    (list 1 2) (list 1 2)) ==> #f
    ```

- If we change an object, is it the same object?

  -- Yes, if we retain the same pointer to the object

- How do we tell if part of an object is *shared* with another?

  -- If we mutate one, see if the other also changes

- Notice: No way to tell the difference without mutation!

One last example...

```
x ==> (3 4)
y ==> (1 2)
```

```
(set-car! x y)
```

x ==> **((1 2) 4)**

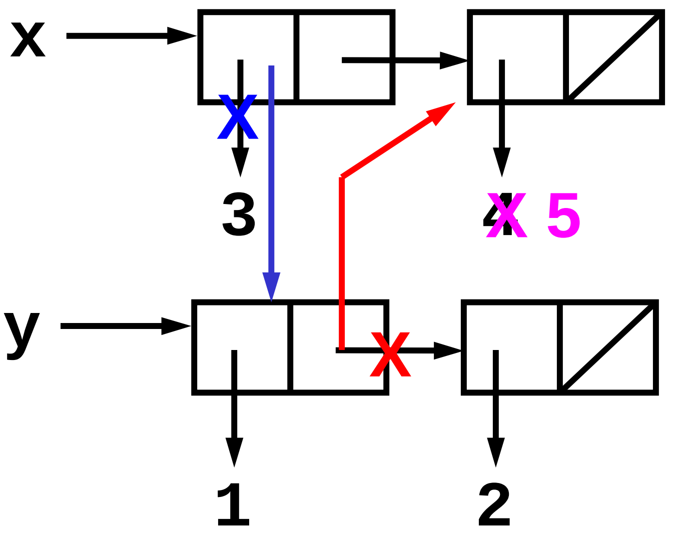
followed by

```
(set-cdr! y (cdr x))
```

x ==> **((1 4) 4)**

```
(set-car! (cdr x) 5)
```

x ==> **((1 5) 5)**



Functional vs Imperative Programming

- Functional programming
  - No assignments
  - As computing mathematical functions
  - No side effects
  - Easy to understand: use the substitution model!
- Imperative programming
  - A style that relies heavily on assignment
  - Introduces new classes of bugs
- This doesn't mean that assignment is evil
  - It sure does complicate things, but:
  - Being able to modify local state is powerful as we will see

Stack Data Abstraction (for recitation)

- constructor:

`(make-stack)`	returns an empty stack

- selectors:

`(top-stack s)`	returns current top element from a stack **`s`**

- operations:

`(push-stack s elt)`	returns a new stack with the element added to the top of the stack
`(pop-stack s)`	returns a new stack with the top element removed from the stack
`(empty-stack? s)`	returns #t if no elements, #f otherwise

Stack Contract

- If **`s`** is a stack, created by **`(make-stack)`** and subsequent stack procedures, where i is the number of **`pushes`** and j is the number of **`pops`**, then

- If $j>i$ then it is an error

- If $j=i$ then **`(empty-stack? s)`** is true, and **`(top-stack s)`** is an error.

- If $j<i$ then **`(empty-stack? s)`** is false, and for any **`val,`**

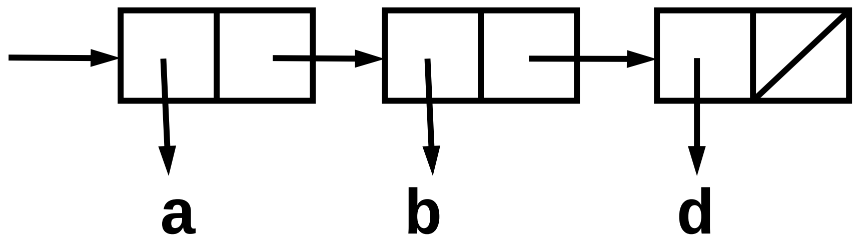
```
(top-stack
  (pop-stack
    (push-stack s val))) = (top-stack s)
```

- If $j<=i$ then for any **`val`**,

```
(top-stack (push-stack s val))) = val
```

Stack Implementation Strategy

- Implement a stack as a list



- We will insert and delete items at the front of the list

Stack Implementation

```
; Stack<A> = List<A>
(define (make-stack) '())

(define (empty-stack? s) ; Stack<A> -> boolean
  (null? s))

(define (push-stack s elt) ; Stack<A>, A -> Stack<A>
  (cons elt s))

(define (pop-stack s) ; Stack<A> -> Stack<A>
  (if (not (empty-stack? s))
      (cdr s)
      (error "stack underflow – delete"))

(define (top-stack s) ; Stack<A> -> A
  (if (not (empty-stack? s))
      (car s)
      (error "stack underflow – top")))
```

Queue Data Abstraction (Non-Mutating)

- constructor:

`(make-queue)`	returns an empty queue

- accessors:

`(front-queue q)`	returns the object at the front of the queue. If queue is empty signals error

- operations:

`(insert-queue q elt)`	returns a new queue with elt at the rear of the queue
`(delete-queue q)`	returns a new queue with the item at the front of the queue removed
`(empty-queue? q)`	tests if the queue is empty

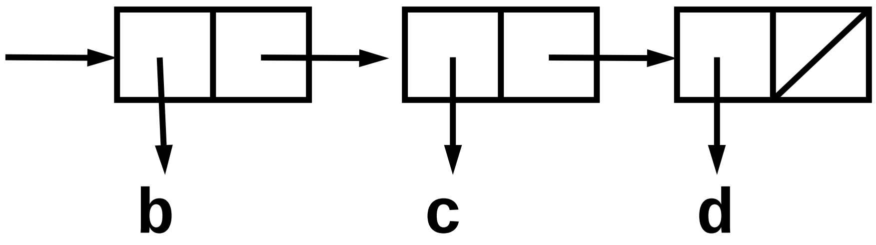
Queue Contract

Given **`q`** is a queue, created by **`(make-queue)`** and subsequent queue procedures, where i is the number of **`inserts`**, and j is the number of **`deletes`**

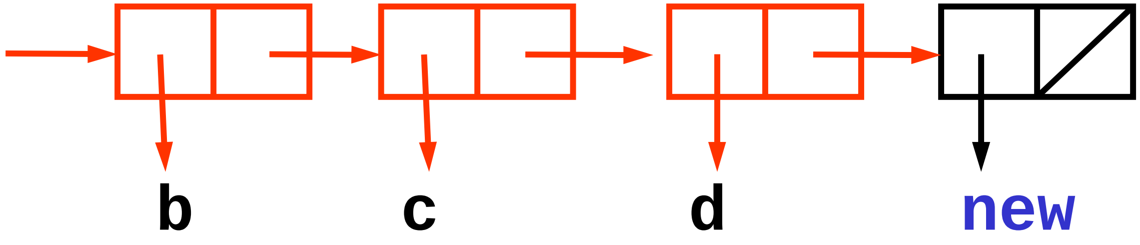
- If $j>i$ then it is an error
- If $j=i$ then **`(empty-queue? q)`** is true, and **`(front-queue q)`** is an error
- If $j<i$ then **`(empty-queue? q)`** is false, and **`(front-queue q)`** is the $(j+1)$th element inserted into the queue

Simple Queue Implementation – pg. 1

- Let the queue simply be a list of queue elements:



- The front of the queue is the first element in the list
- To insert an element at the tail of the queue, we need to "copy" the existing queue onto the front of the **new** element:



Simple Queue Implementation – pg. 2

```
(define (make-queue) '())

(define (empty-queue? q) (null? q)); Queue<A> -> boolean

(define (front-queue q) ; Queue<A> -> A
  (if (not (empty-queue? q))
      (car q)
      (error "front of empty queue:" q)))

(define (delete-queue q) ; Queue<A> -> Queue<A>
  (if (not (empty-queue? q))
      (cdr q)
      (error "delete of empty queue:" q)))

(define (insert-queue q elt) ; Queue<A>, A -> Queue<A>
  (if (empty-queue? q)
      (cons elt '())
      (cons (car q) (insert-queue (cdr q) elt))))
```

Simple Queue - Efficiency

- How efficient is the simple queue implementation?
  - For a queue of length *n*
    - – Time required – number of iterations?
    - – Space required – number of pending operations?

- **`front-queue, delete-queue:`**
  - Time: Constant
  - Space: Constant

- **`insert-queue:`**
  - Time: Linear
  - Space: Linear

Limitations in our Queue

- Queue does not have ***identity***

```
(define q (make-queue))
q ==> ()

(insert-queue q 'a) ==> (a)
q ==> ()

(set! q (insert-queue q 'b))
q ==> (b)
```

Queue Data Abstraction (Mutating)

- constructor:

`(make-queue)`	returns an empty queue

- accessors:

`(front-queue q)`	returns the object at the front of the queue. If queue is empty signals error

- mutators:

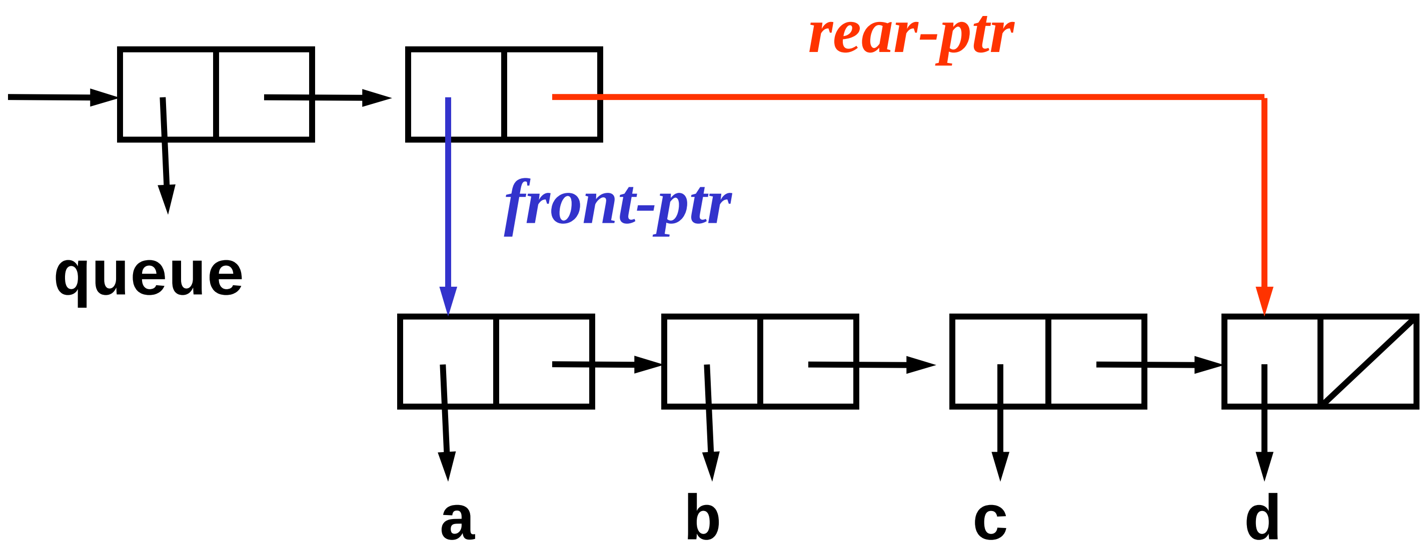
`(insert-queue! q elt)`	inserts the elt at the rear of the queue and returns the modified queue
`(delete-queue! q)`	removes the elt at the front of the queue and returns the modified queue

- operations:

`(queue? q)`	tests if the object is a queue
`(empty-queue? q)`	tests if the queue is empty

Better Queue Implementation – pg. 1

- We'll attach a type tag as a defensive measure
- Maintain queue ***identity***
- Build a structure to hold:
  - a list of items in the queue
  - a pointer to the front of the queue
  - a pointer to the rear of the queue



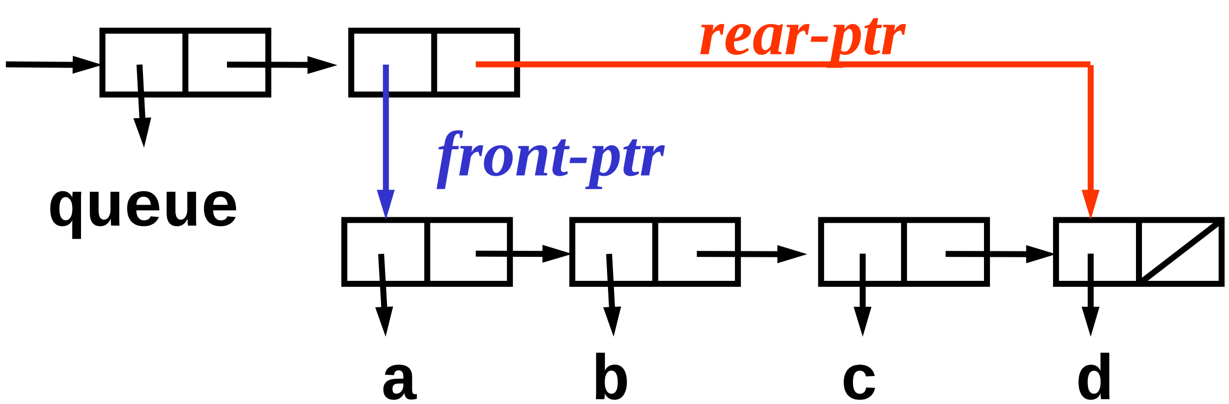
Queue Helper Procedures

- Hidden inside the abstraction

```
(define (front-ptr q) (cadr q))
(define (rear-ptr q)  (cddr q))

(define (set-front-ptr! q item)
  (set-car! (cdr q) item))

(define (set-rear-ptr! q item)
  (set-cdr! (cdr q) item))
```



Better Queue Implementation – pg. 2

```
(define (make-queue)
  (cons 'queue (cons '() '())))

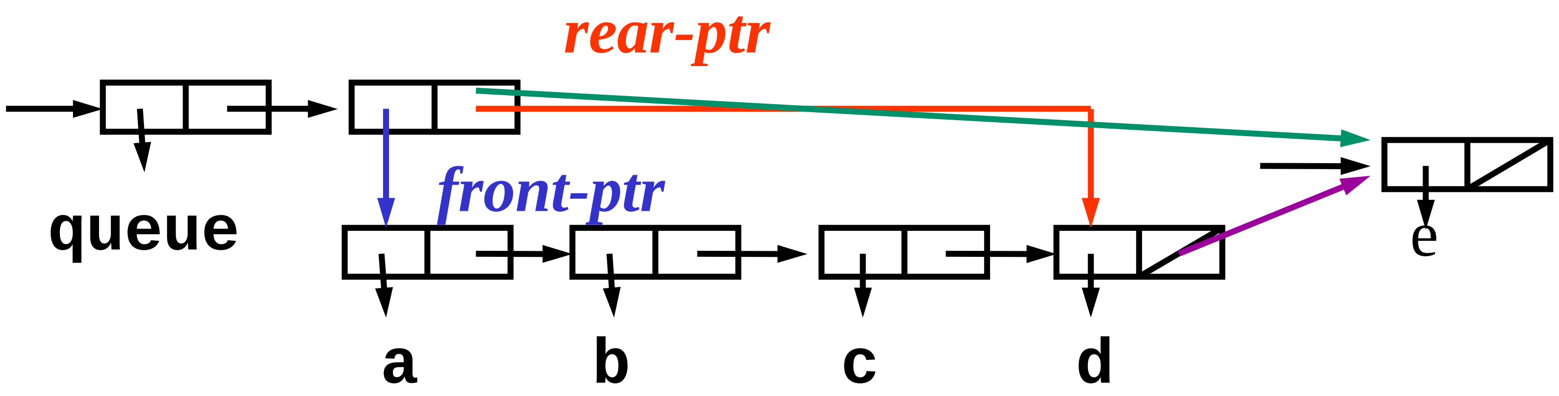
(define (queue? q) ; anytype -> boolean
  (and (pair? q) (eq? 'queue (car q))))

(define (empty-queue? q) ; Queue<A> -> boolean
  (if (queue? q)
      (null? (front-ptr q))
      (error "object not a queue:" q)))

(define (front-queue q) ; Queue<A> -> A
  (if (not (empty-queue? q))
      (car (front-ptr q))
      (error "front of empty queue:" q)))
```

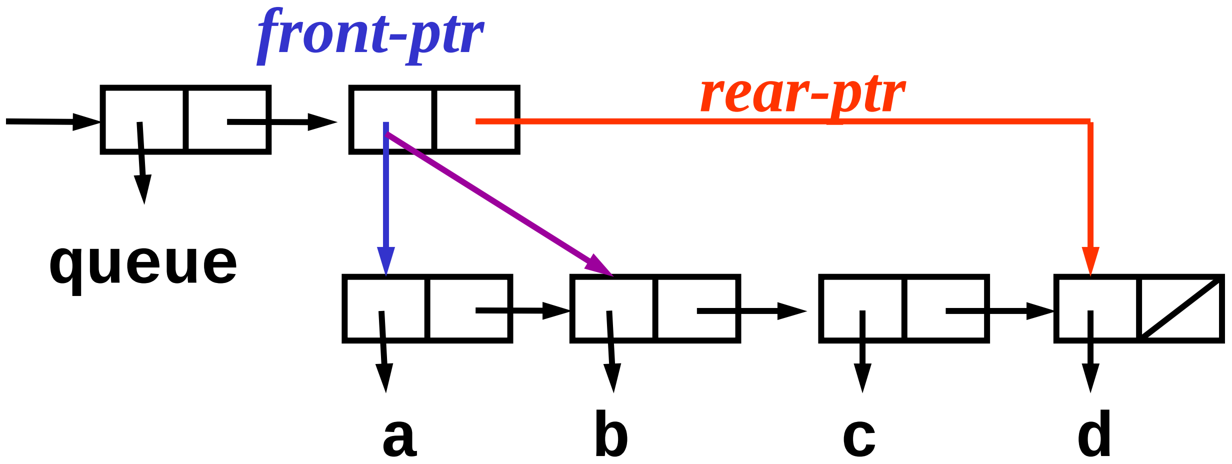
Queue Implementation – pg. 3

```
(define (insert-queue! q elt); Queue<A>, A -> Queue<A>
  (let ((new-pair (cons elt '())))
   (cond ((empty-queue? q) (set-front-ptr! q new-pair)
                           (set-rear-ptr! q new-pair))
         (else (set-cdr! (rear-ptr q) new-pair)
               (set-rear-ptr! q new-pair)))
   q)))
```



Queue Implementation – pg. 4

```
(define (delete-queue! q) ; Queue<A> -> Queue<A>
  (if (not (empty-queue? q))
      (set-front-ptr! q (cdr (front-ptr q)))
      (error "delete of empty queue:" q))
  q)
```



Mutating Queue - Efficiency

- How efficient is the mutating queue implementation?
  - For a queue of length *n*
    - Time required -- number of iterations?
    - Space required -- number of pending operations?

- **`front-queue, delete-queue!:`**
  - Time: Constant
  - Space: Constant

- **`insert-queue!:`**
  - Time: $T(n)$ = ***Constant***
  - Space: $S(n)$ = ***Constant***

Summary - Catch your breath

- Built-in mutators which operate by **side-effect**
  - **`set!           (special form)`**
  - **`set-car!       ; Pair, anytype -> undef`**
  - **`set-cdr!       ; Pair, anytype -> undef`**

- Extend our notion of data abstraction to include mutators

- Mutation is a powerful idea
  - enables new and efficient data structures
  - can have surprising side effects
  - breaks our model of "functional" programming (substitution model)

Can you figure out why this code works?

```
(define make-counter
   (lambda (n)
     (lambda () (set! n (+ n 1))
                n   )))

(define ca (make-counter 0))
(ca) ==> 1
(ca) ==> 2   ; not functional programming!
(define cb (make-counter 0))
(cb) ==> 1
(ca) ==> 3   ; ca and cb are independent
```

Need a new model of mutation for closures.

What the Environment Model is:

- A precise, completely mechanical description:
  - name-rule — looking up the value of a variable
  - define-rule — creating a new definition of a var
  - set!-rule — changing the value of a variable
  - lambda-rule — creating a procedure
  - application — applying a procedure

- Enables analyzing more complex scheme code:
  - Example: **make-counter**

- Basis for implementing a scheme interpreter
  - for now: draw EM state with boxes and pointers
  - later on: implement with code

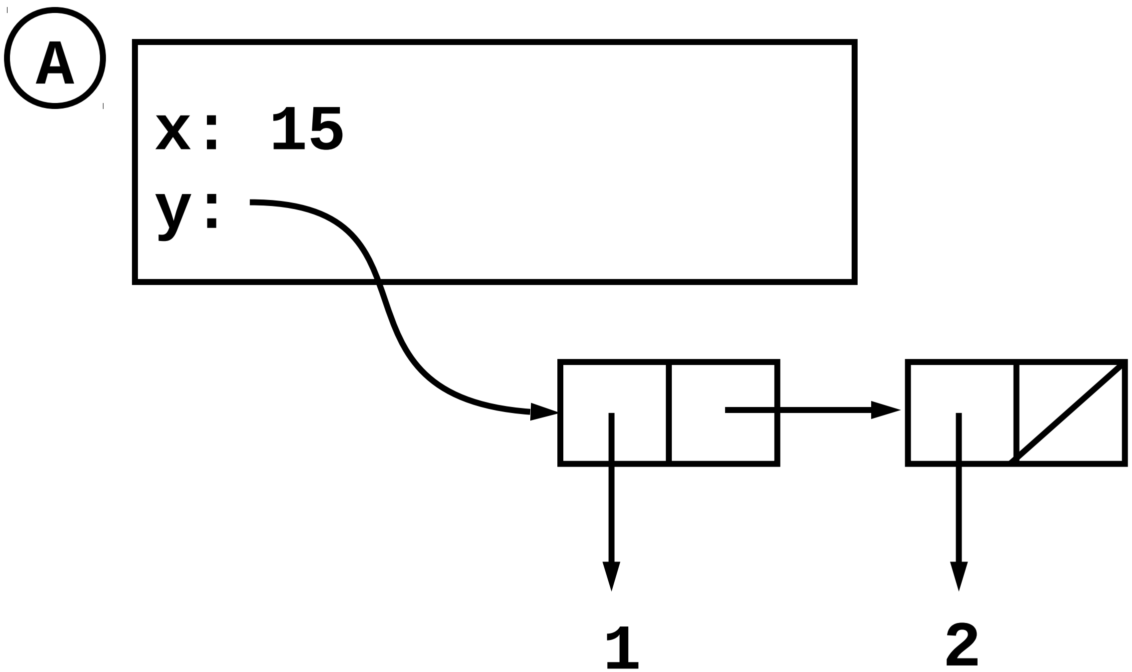
A shift in viewpoint

- As we introduce the environment model, we are going to shift our viewpoint on computation
- Variable:
  - OLD – name for value
  - NEW – place into which one can store things
- Procedure:
  - OLD – functional description
  - NEW – object with inherited context
- Expressions
  - Now **only** have meaning with respect to an environment

Frame: a table of bindings

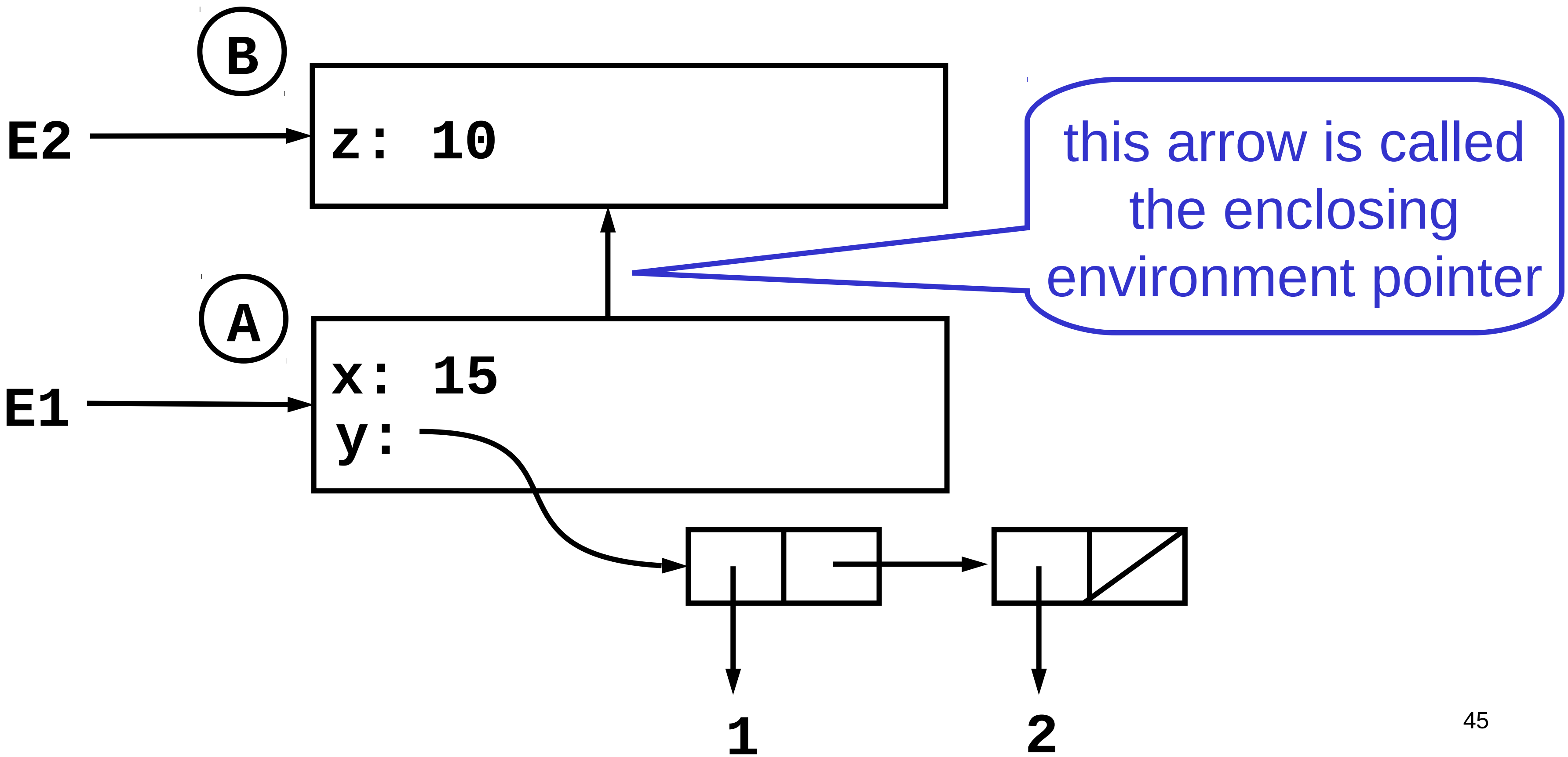
- Binding: a pairing of a name and a value

  Example: **x** is bound to **15** in frame A
  y is bound to **(1 2)** in frame A
  the value of the variable **x** in frame A is 15



Environment: a sequence of frames

- Environment E1 consists of frames A and B
- Environment E2 consists of frame B only
  - A frame may be shared by multiple environments



Evaluation in the environment model

- All evaluation occurs with respect to an environment
  - The current environment changes when the interpreter applies a procedure

- The top environment is called the global environment (GE)
  - Only the GE has no enclosing environment
- To evaluate a combination
  - Evaluate the subexpressions *in the current environment*
  - Apply the value of the first to the values of the rest

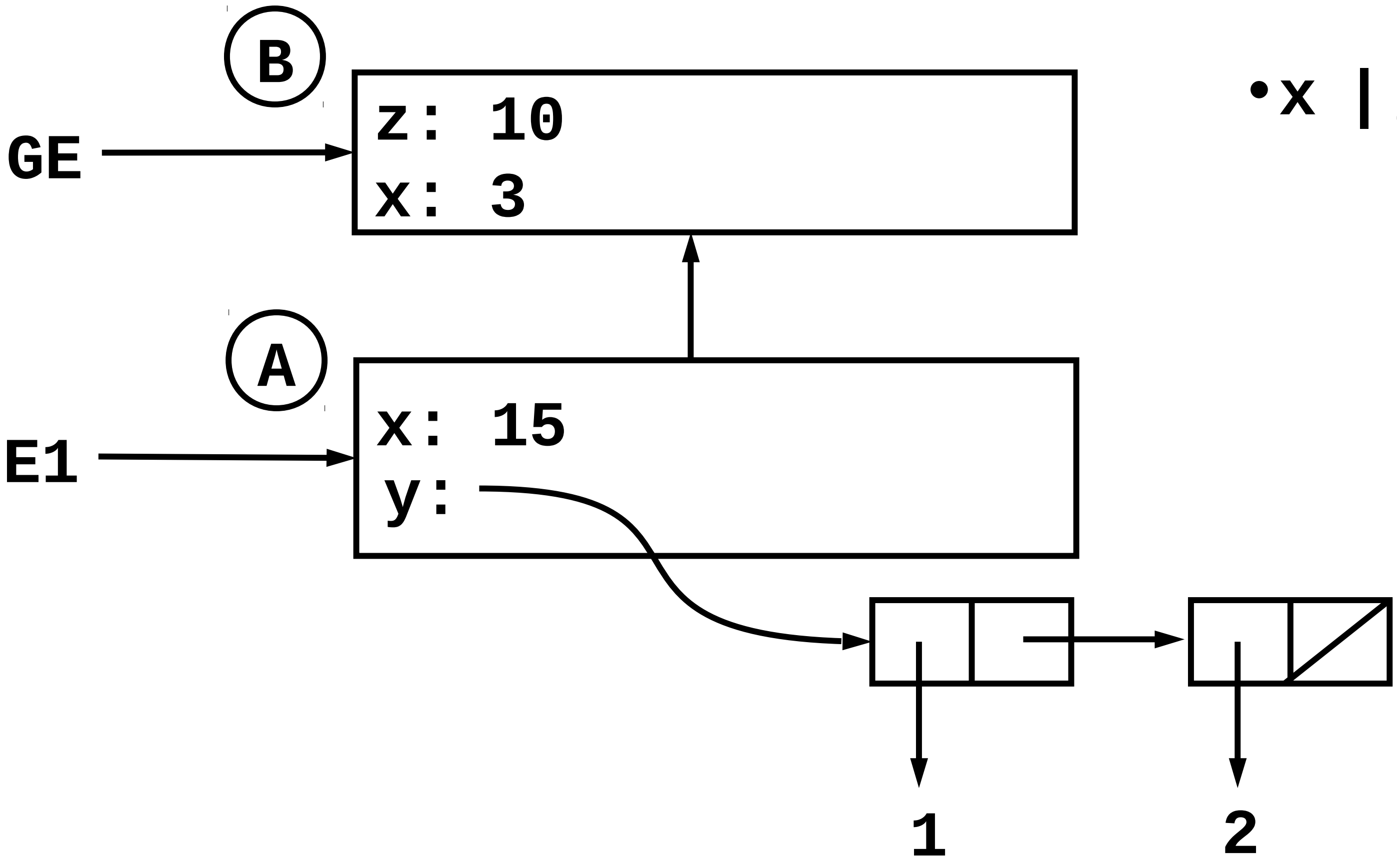
Name-rule

- A name X evaluated in environment E gives
  the value of X in *the first frame of E where X is bound*

  - `z | GE ==>`   `z | E1 ==>`   `x | E1 ==>`

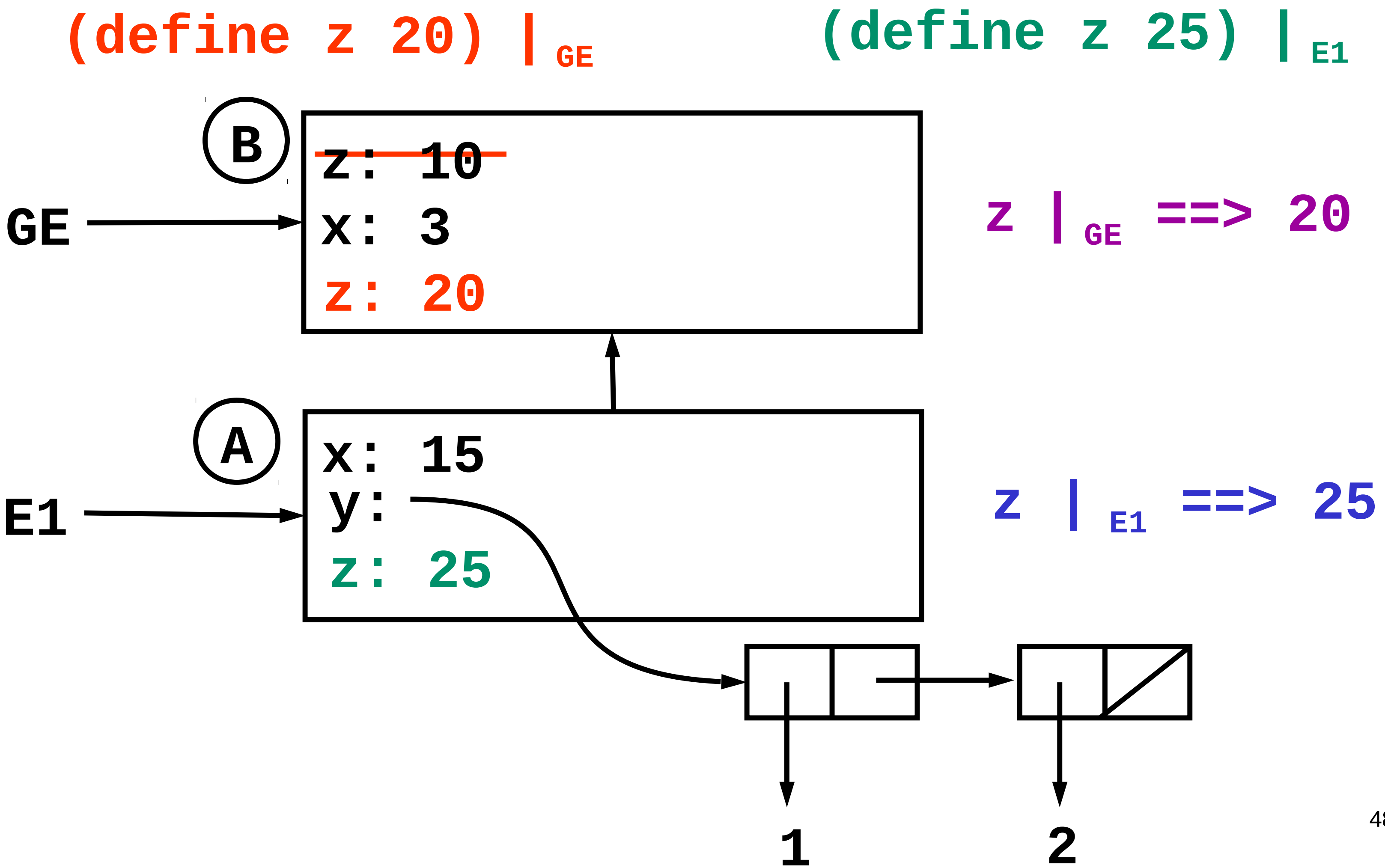
- In E1, the binding of x in frame A shadows the binding of x in B

  - `x | GE ==> 3`



Define-rule

- A define special form evaluated in environment E creates or replaces a binding in the first frame of E

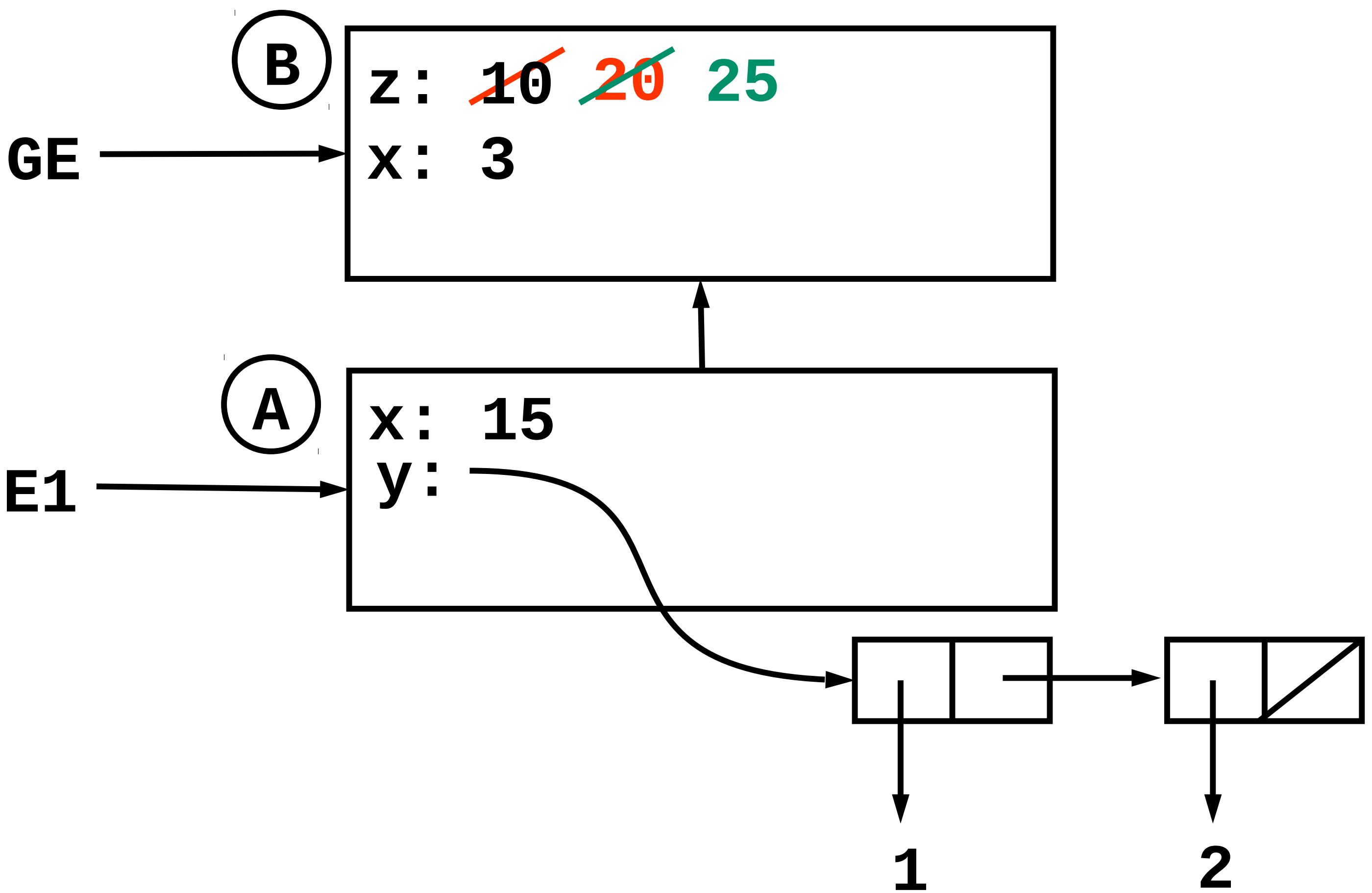


Set!-rule

- A set! of variable X evaluated in environment E changes the binding of X in the first frame of E where X is bound

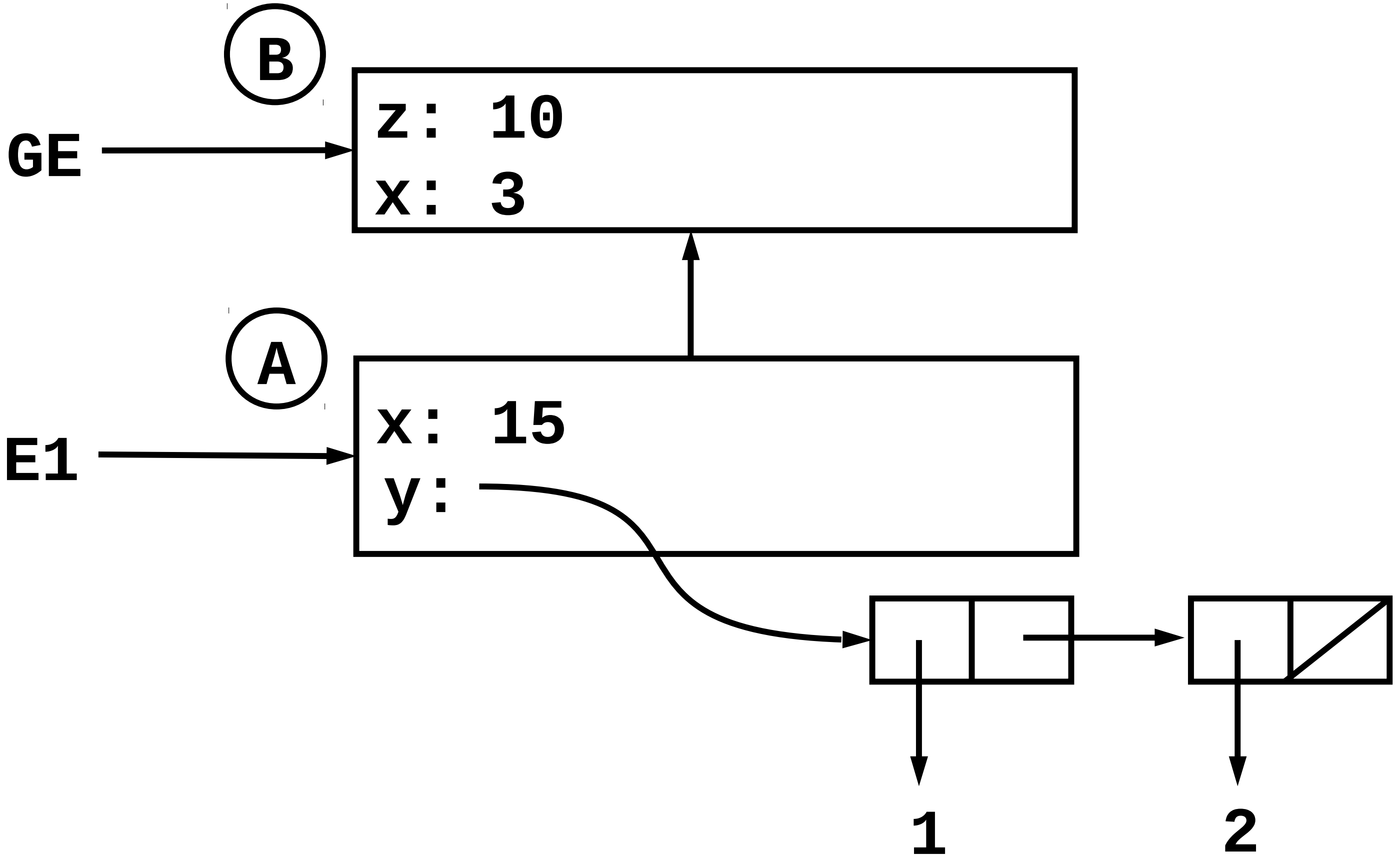
(set! z 20) | GE

(set! z 25) | E1



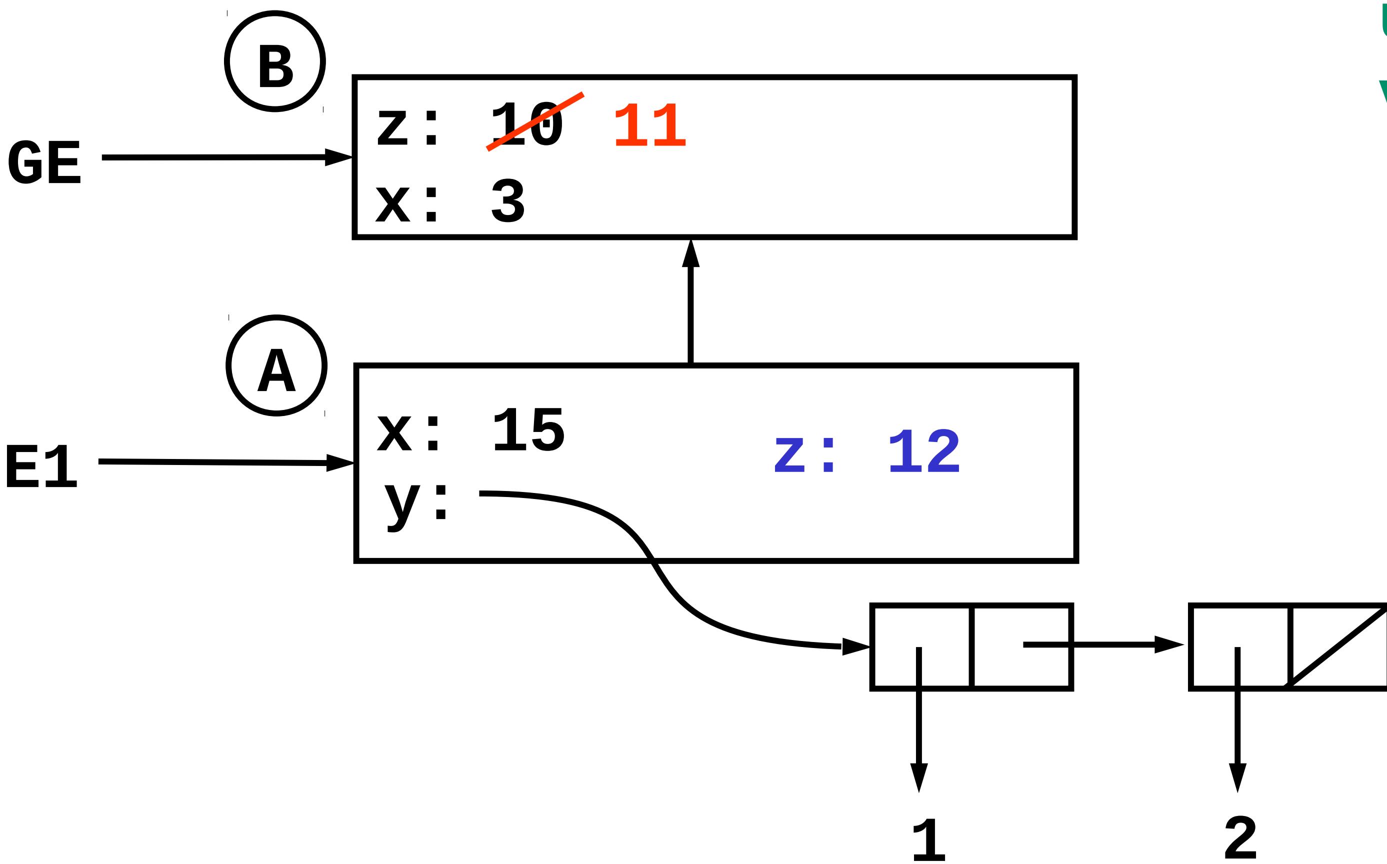
Your turn: evaluate the following in order

```
(+ z 1) |E1                    ==>          11
(set! z (+ z 1)) |E1           (modify EM)
(define z (+ z 1)) |E1         (modify EM)
(set! y (+ z 1)) |GE           (modify EM)
```

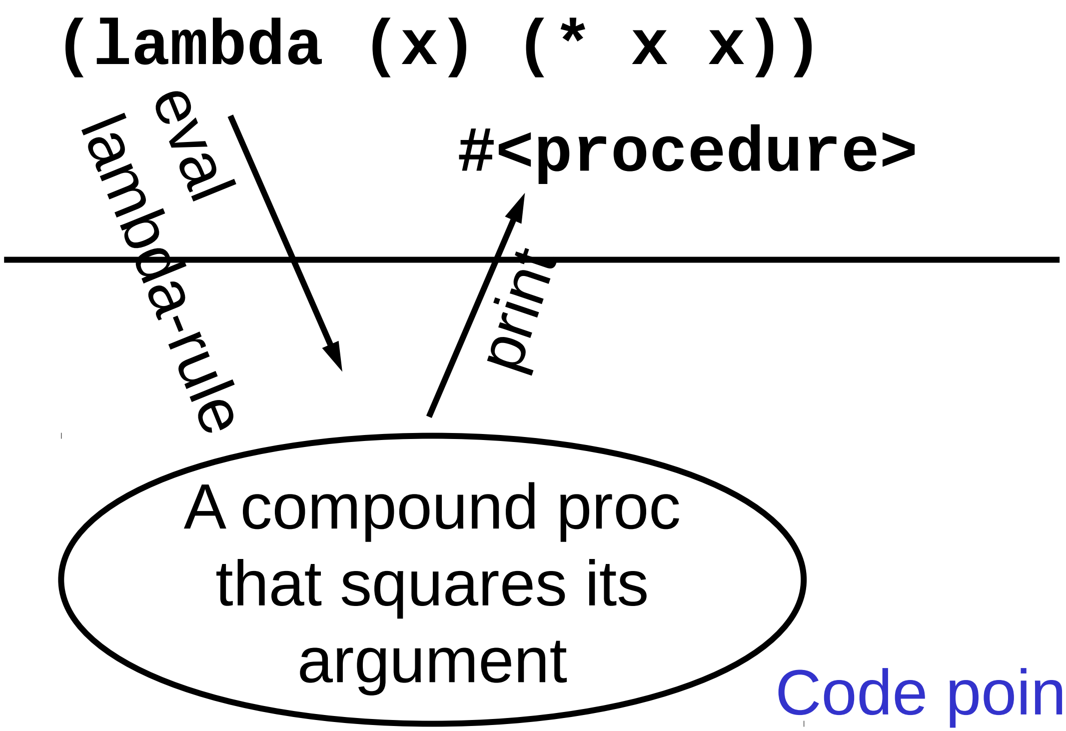


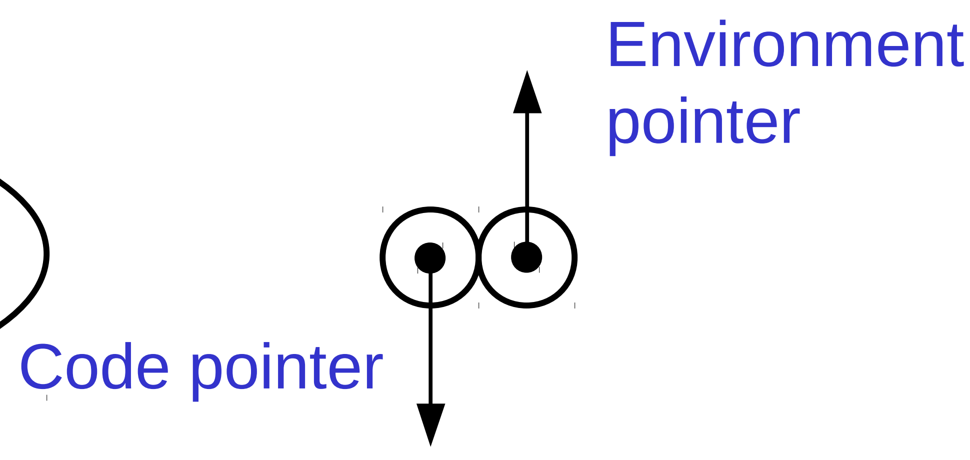
Your turn: evaluate the following in order

```
(+ z 1) |E1                        ==>                11
(set! z (+ z 1)) |E1               (modify EM)
(define z (+ z 1)) |E1             (modify EM)
(set! y (+ z 1)) |GE               (modify EM)        Error: unbound variable: y
```



Double bubble: how to draw a procedure



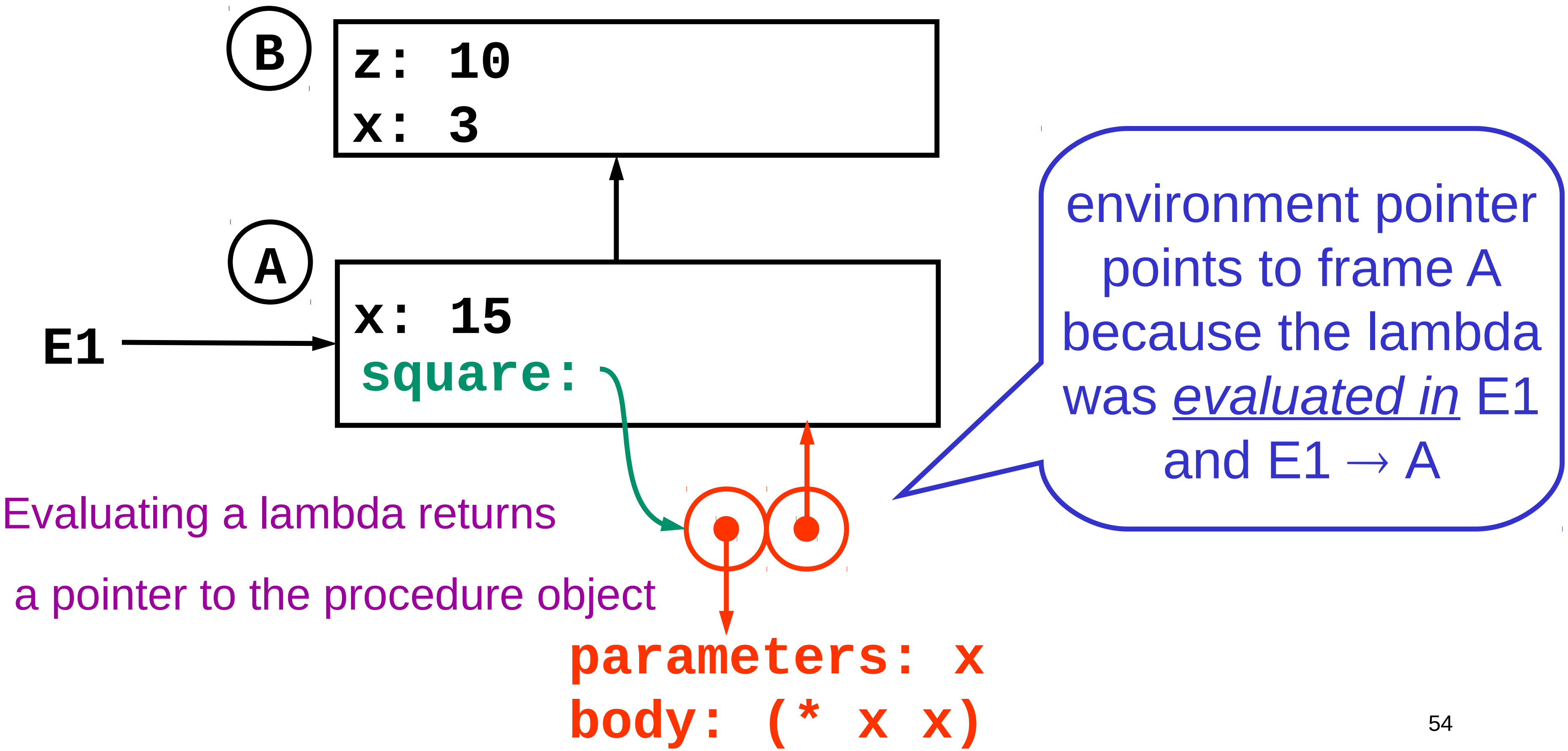



```
parameters: x
body: (* x x)
```

Lambda-rule

- A lambda special form evaluated in environment E creates a procedure whose environment pointer is E

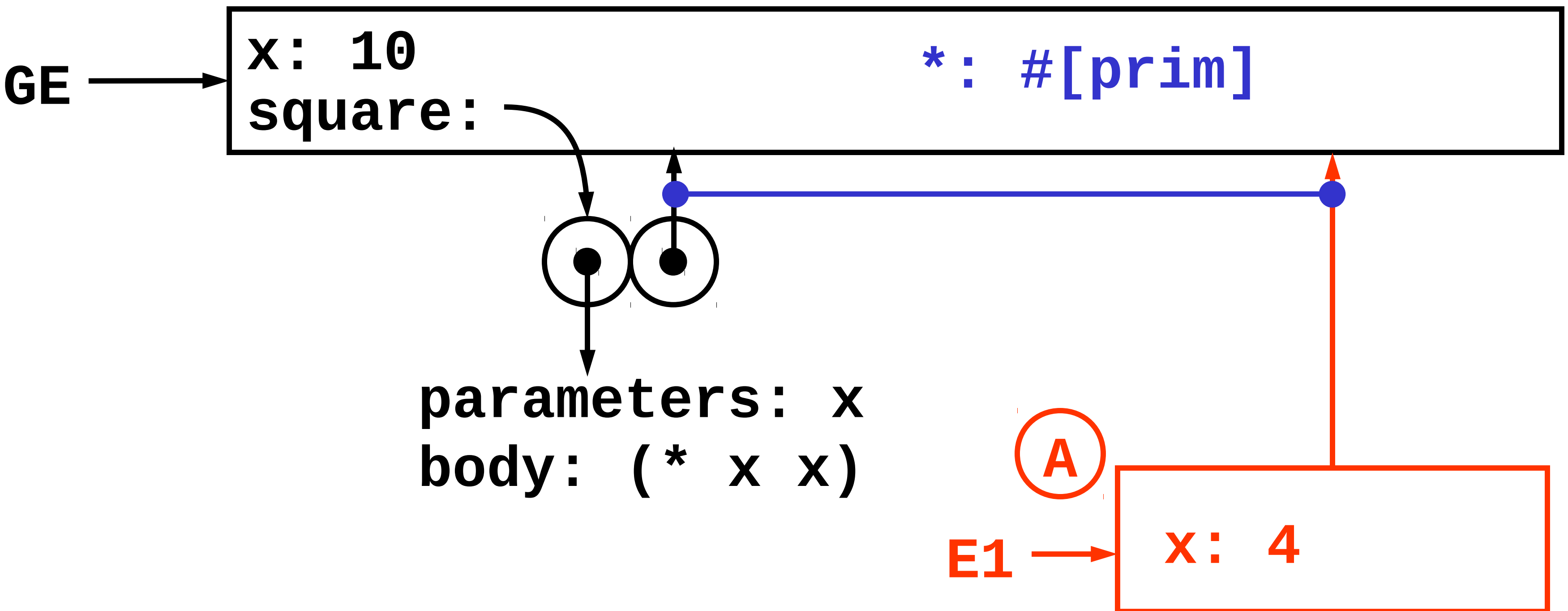
```
(define square (lambda (x) (* x x))) | E1
```



To apply a compound procedure P to arguments:

1. Create a new frame A
2. Make A into an environment E:
   A's enclosing environment pointer goes to *the same frame as the environment pointer of P*
3. In A, bind the parameters of P to the argument values
4. Evaluate the body of P with E as the current environment

(square 4) | GE



square | GE ==> #<proc>

(* x x) | E1 ==> 16

* | E1 ==> #[prim]

x | E1 ==> 4

Lessons from this example

- EM doesn't show the complete state of the interpreter
  - missing the stack of pending operations

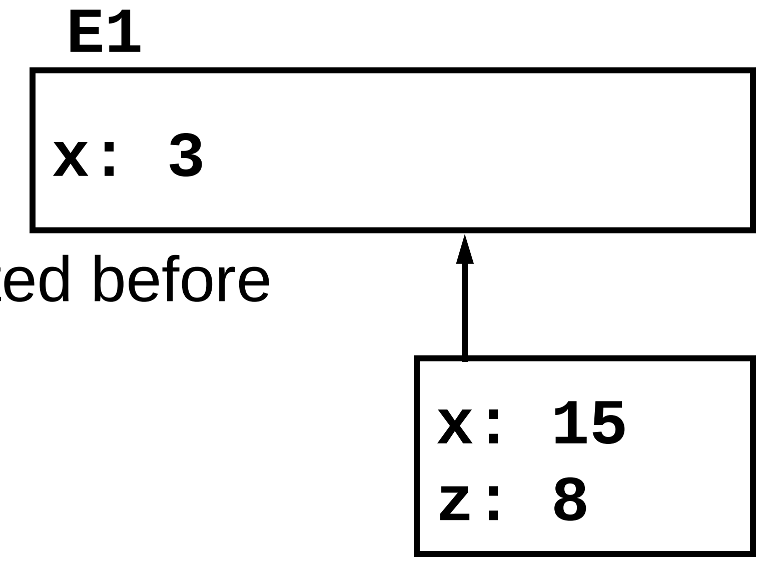
- The GE contains all standard bindings (*****, **cons**, etc)
  - omitted from EM drawings

- Useful to link environment pointer of each frame
  to the procedure that created it

Let special form

- A let expression evaluated in environment E evaluates the values for the new variables, and then drops a new frame whose parent frame is E, binding them to the given names

```
(let ((x 15)
      (z (+ x 5)))
     (* z 2)) | E1
```



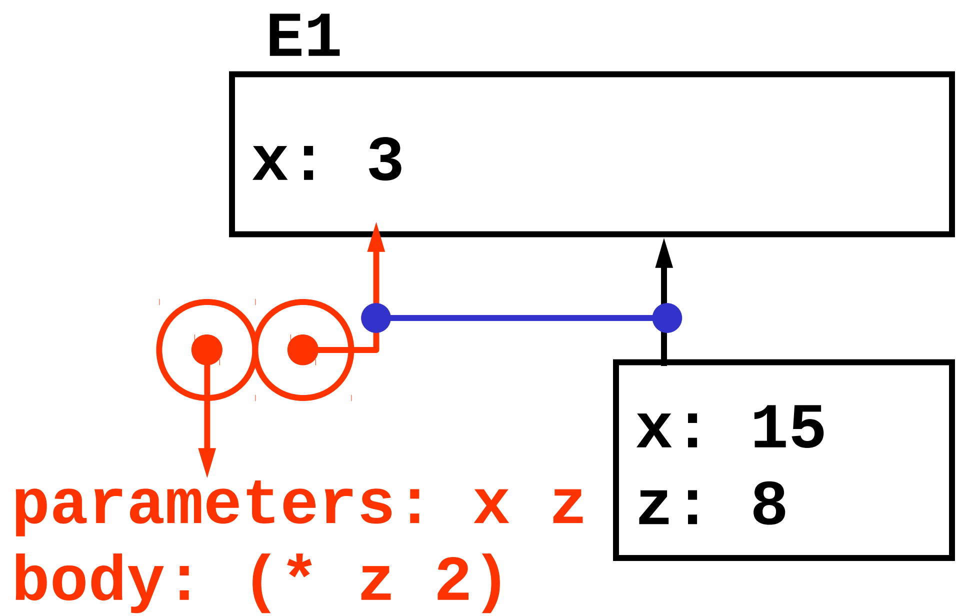
- The binding values are evaluated before the new frame is created.
- The body is evaluated in the new environment
- Sounds familiar....

=> 16

Let special form

- A let expression evaluated in environment E evaluates the values for the new variables, and then drops a new frame whose parent frame is E, binding them to the given names

```
(let ((x 15)
      (z (+ x 5)))
    (* z 2)) | E1
```



- Hidden lambda!

=> 16

```
( (lambda (x z) (* z 2))  15   (+ x 5)  )
```

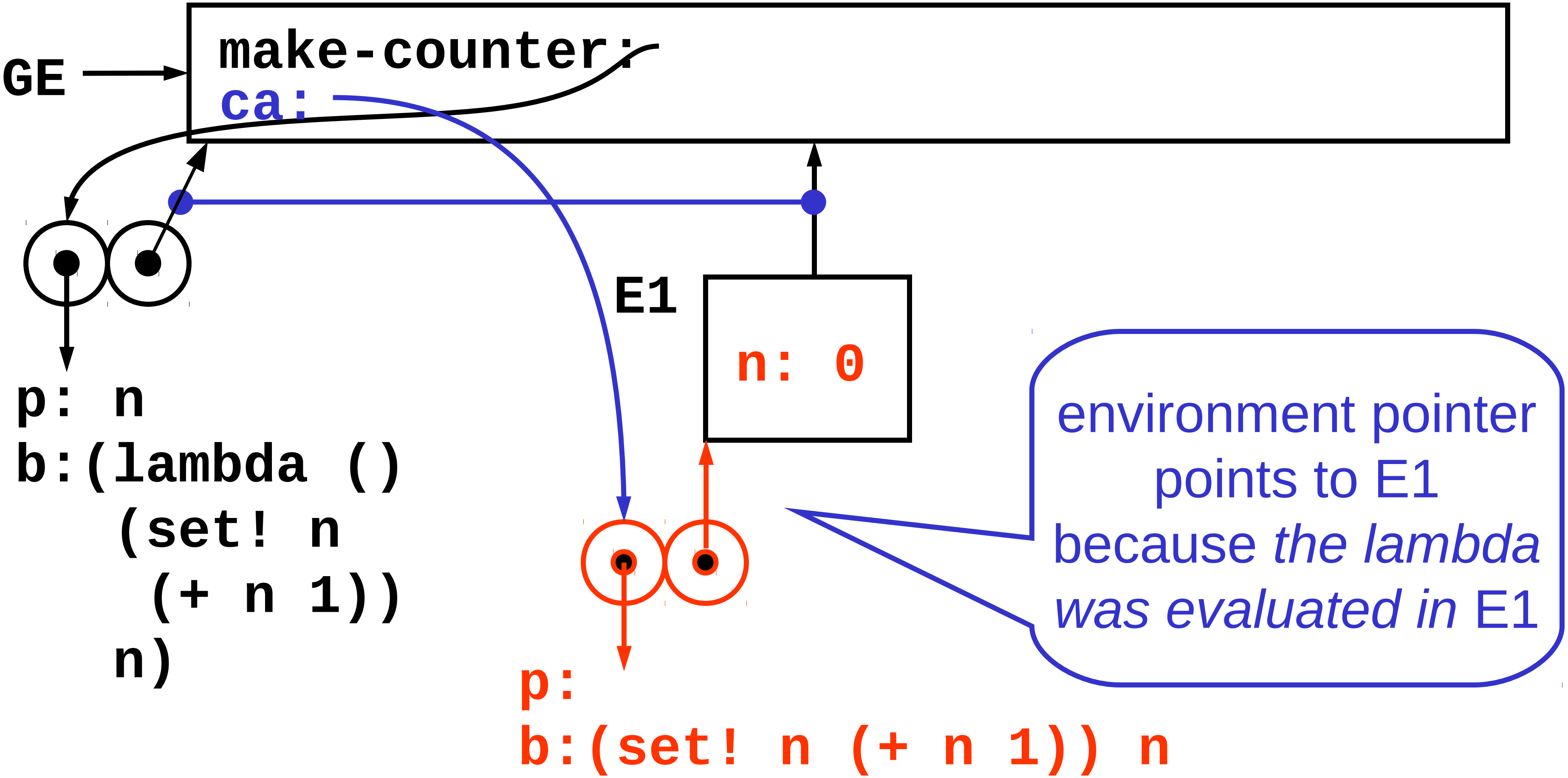
Example: make-counter

- Counter: something which counts up from a number

```
(define make-counter
  (lambda (n)
    (lambda () (set! n (+ n 1))
               n
    )))

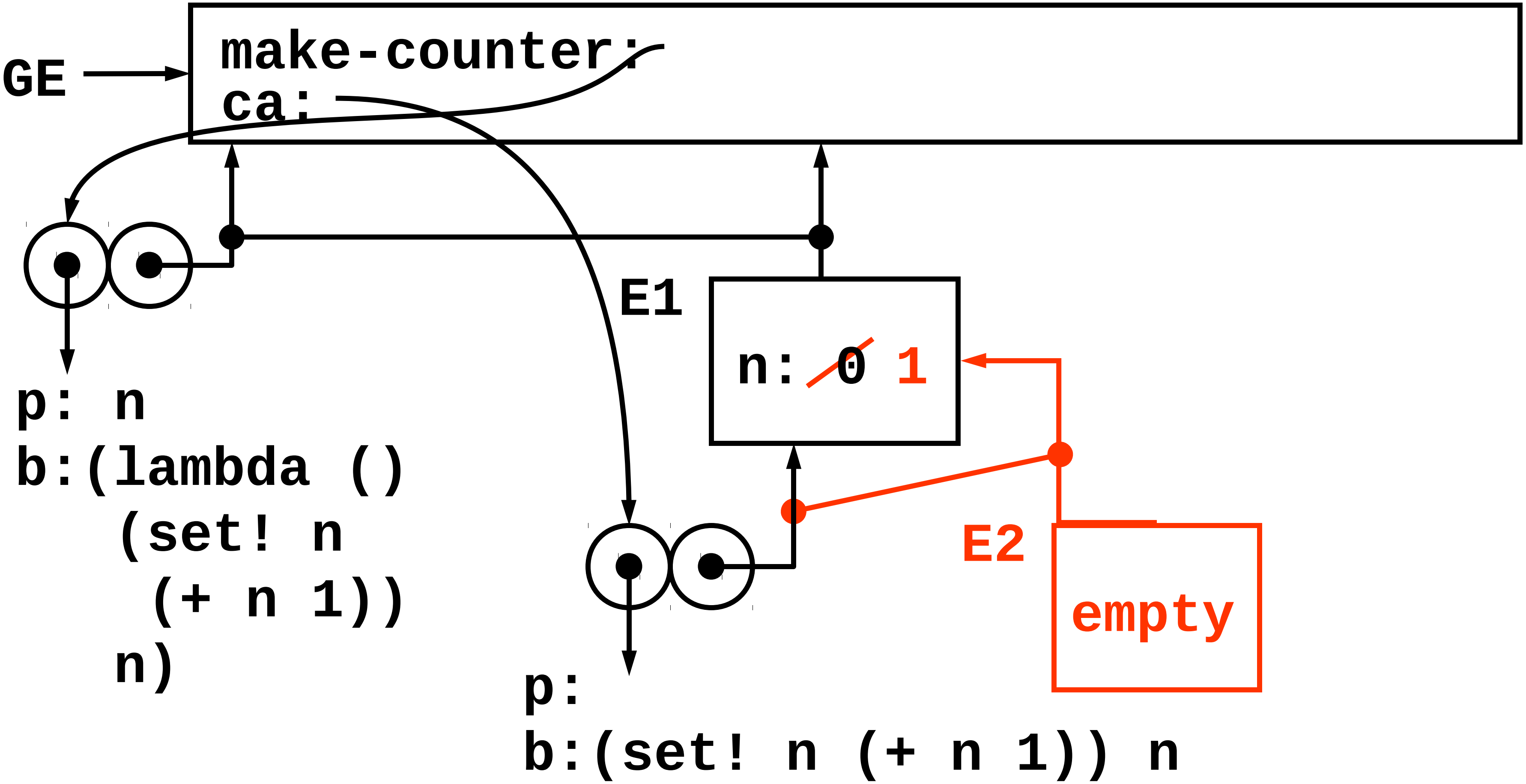
(define ca (make-counter 0))
(ca) ==> 1
(ca) ==> 2 ; not functional programming
(define cb (make-counter 0))
(cb) ==> 1
(ca) ==> 3
(cb) ==> 2  ; ca and cb are independent
```

(define ca (make-counter 0)) |$_{GE}$

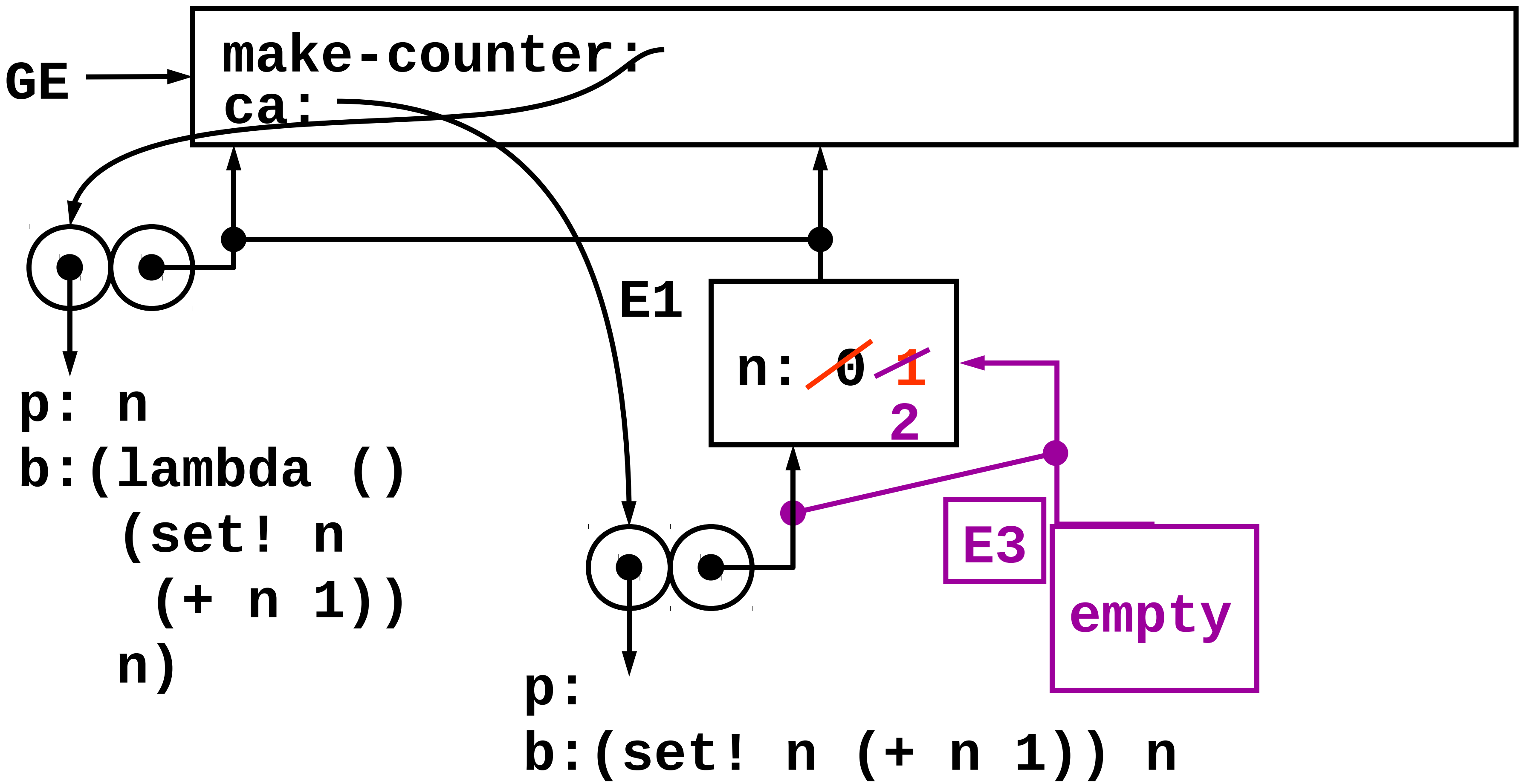


(lambda () (set! n (+ n 1)) n) |$_{E1}$

(ca) | $_{GE}$ ==> 1

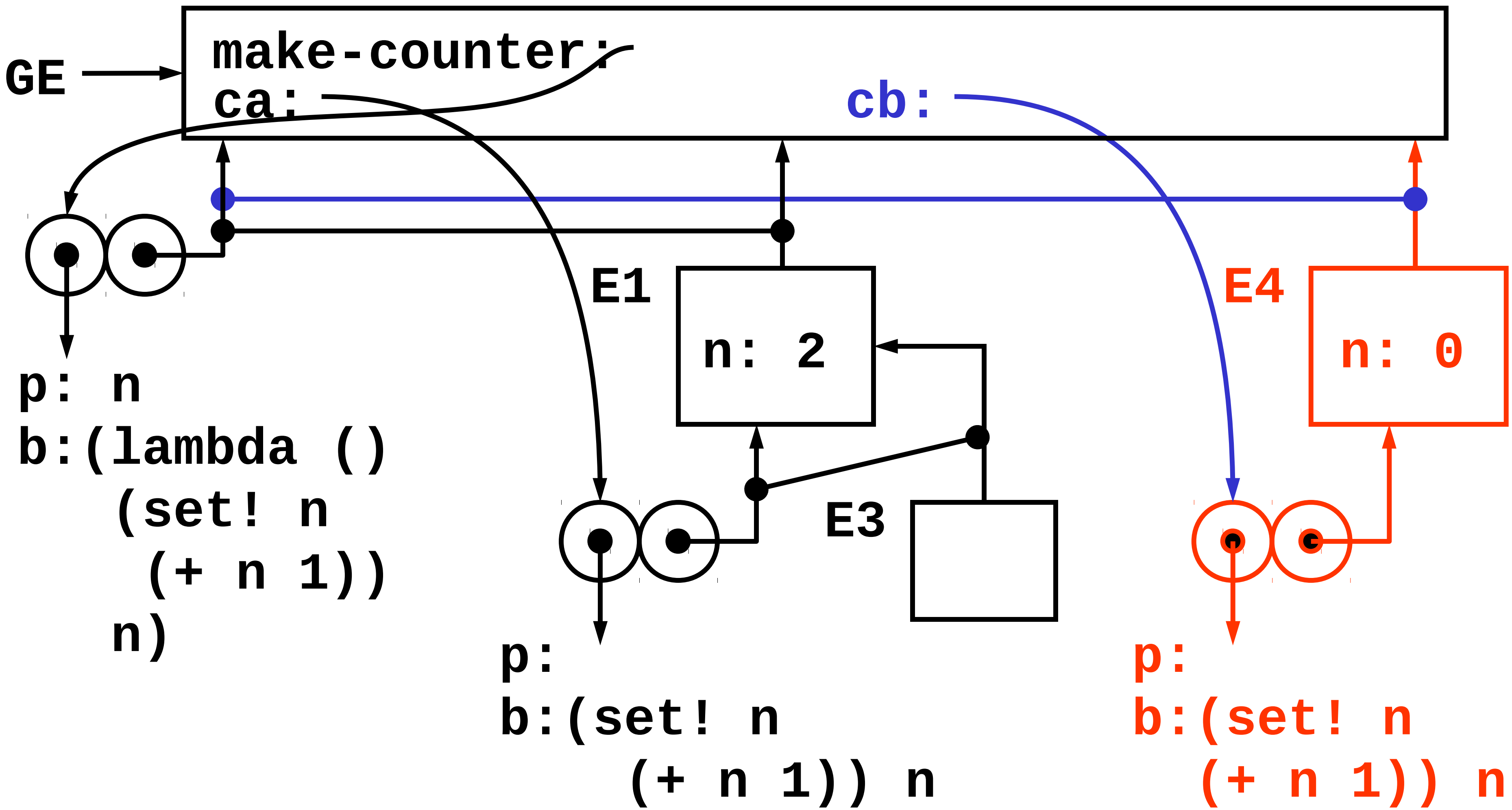


(set! n (+ n 1)) | $_{E2}$ n | $_{E2}$ ==> 1 (ca) |$_{GE}$ ==> 2



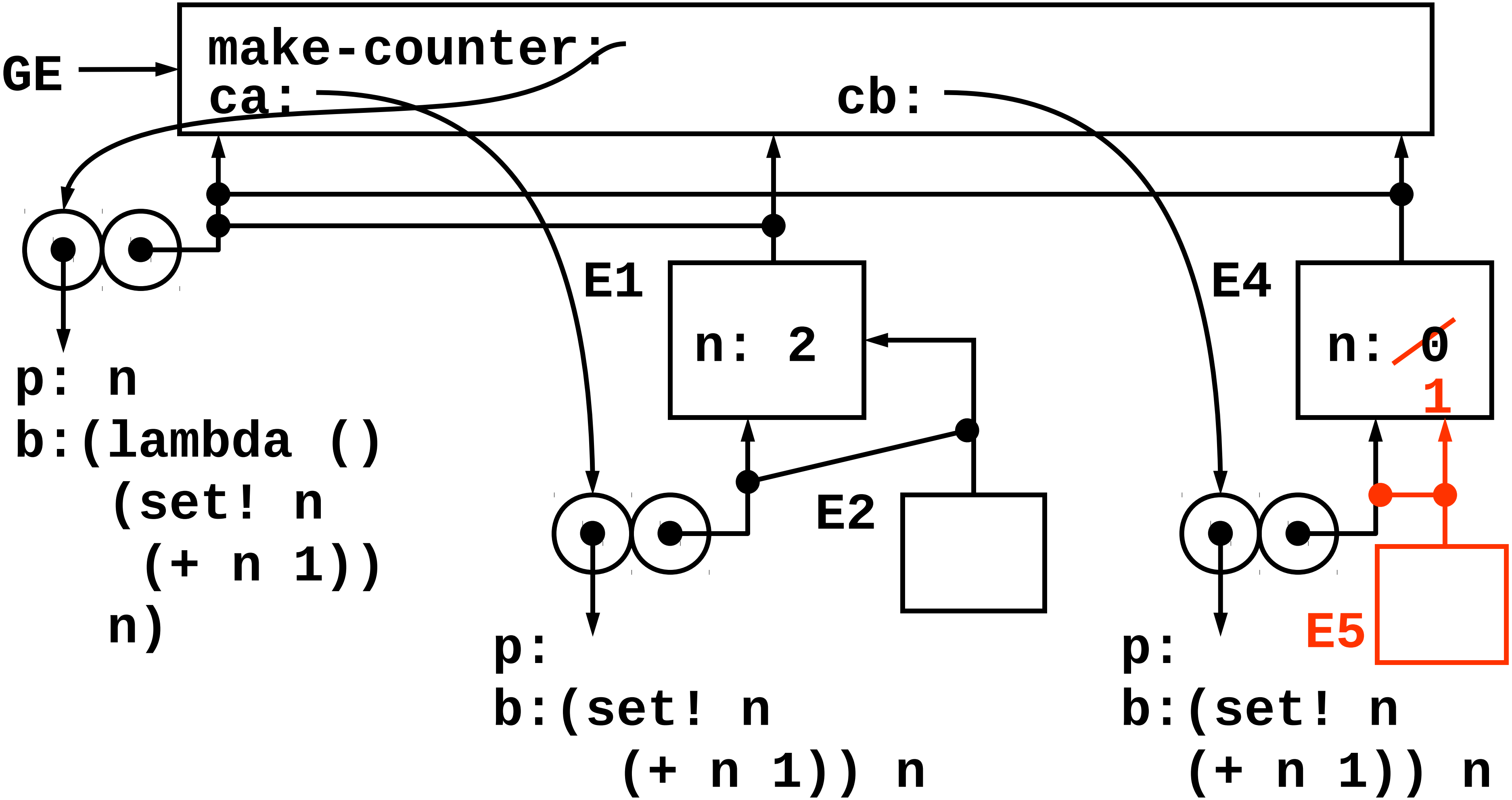
(set! n (+ n 1)) |$_{E3}$ n |$_{E3}$ ==> 2

(define cb (make-counter 0)) |$_{GE}$



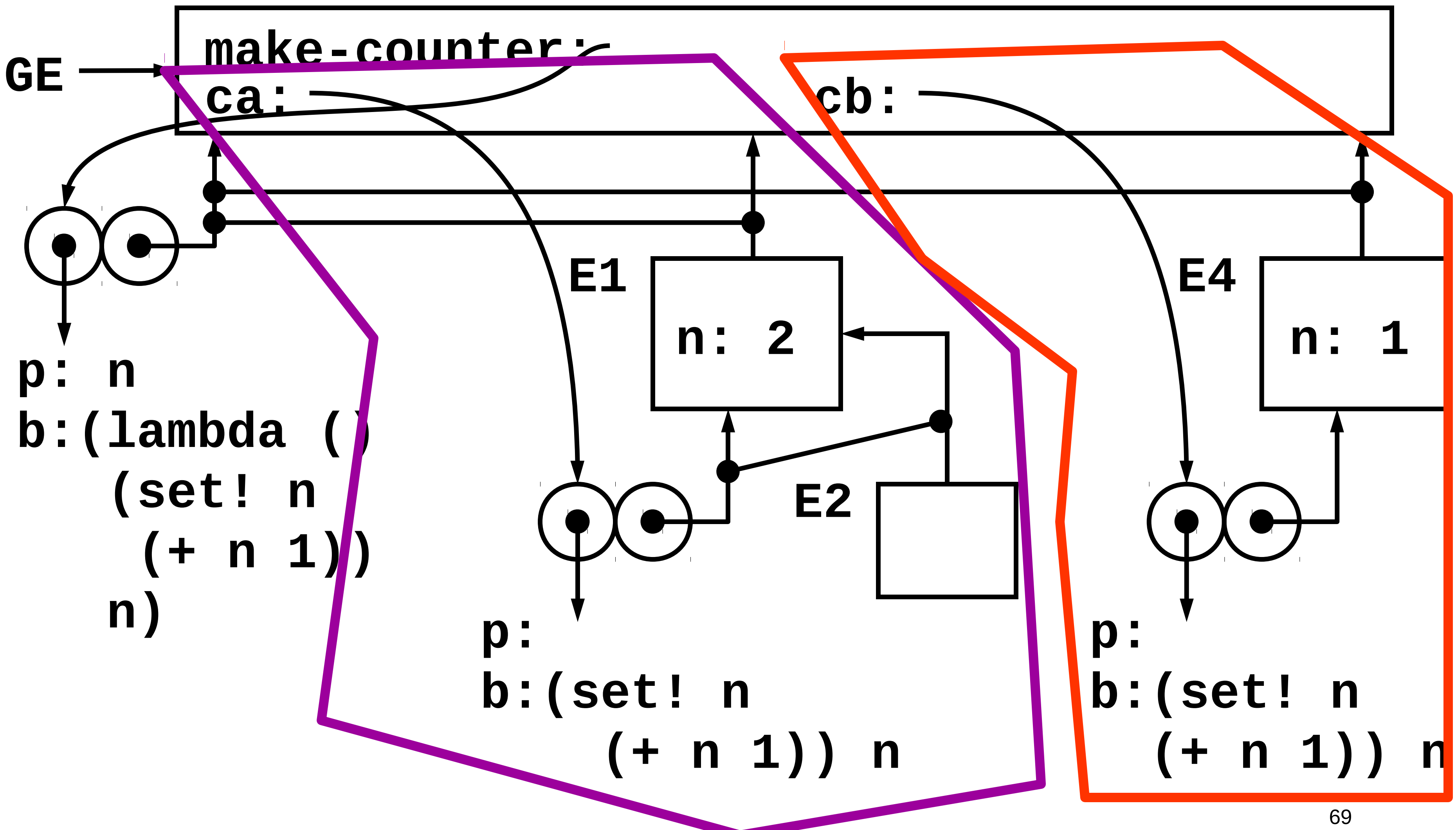
(lambda () (set! n (+ n 1)) n) |$_{E4}$

(cb) | GE ==> 1



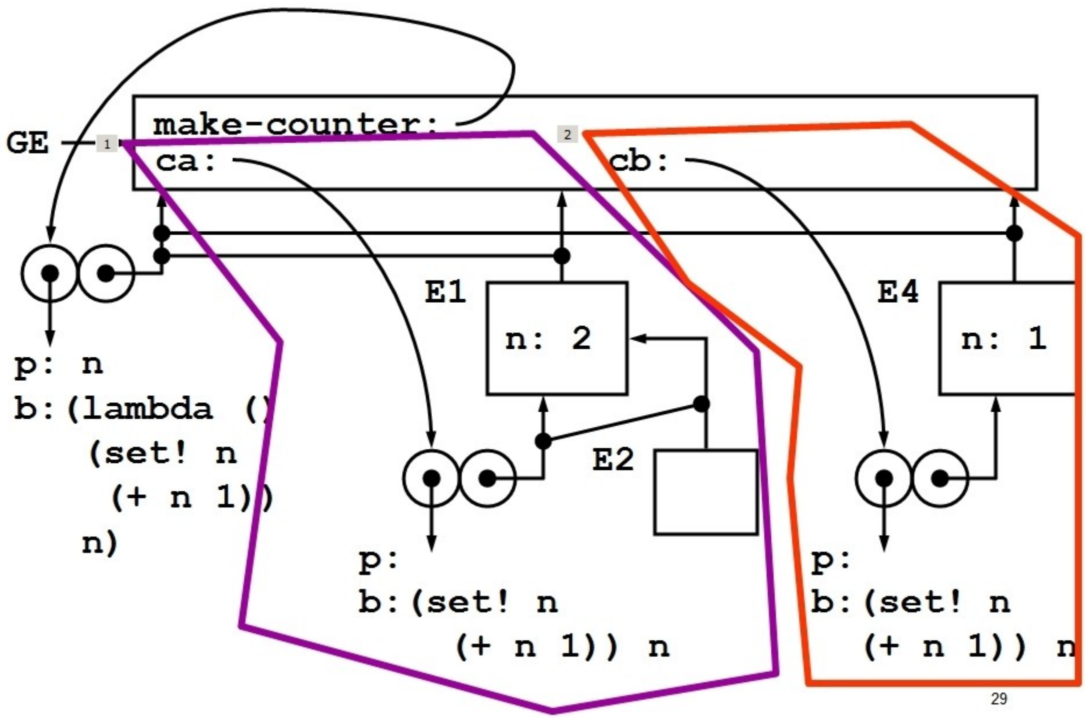
GE
make-counter:
ca:
cb:
E1
n: 2
E2
E4
n: 0
1
E5
p: n
b:(lambda ()
(set! n
(+ n 1))
n)
p:
b:(set! n
(+ n 1)) n
p:
b:(set! n
(+ n 1)) n

Capturing state in local frames & procedures



Lessons from the `make-counter` example

- Environment diagrams get complicated very quickly
  - Rules are meant for the computer to follow, not to help humans
- A lambda inside a procedure body captures the frame that was active when the lambda was evaluated
  - this effect can be used to store local state



Recitation Time!