Continuations

6.037 - Structure and Interpretation of Computer Programs

Mike Phillips <mpp>

Massachusetts Institute of Technology

Lecture 7A

(define the-cons (cons 1 #f))
(set-cdr! the-cons the-cons)

```
(define the-cons (cons 1 #f))
(set-cdr! the-cons the-cons)
```

```
(define (run-in-circles l)
  (+ (run-in-circles (cdr l))))
```

```
(define the-cons (cons 1 #f))
(set-cdr! the-cons the-cons)
```

```
(define (run-in-circles l)
  (+ (run-in-circles (cdr l))))
```

```
(define the-cons (cons 1 #f))
(set-cdr! the-cons the-cons)
```

```
(define (run-in-circles l)
  (+ (run-in-circles (cdr l))))
```

```
(define the-cons (cons 1 #f))
(set-cdr! the-cons the-cons)
```

```
(define (run-in-circles l)
  (+ (run-in-circles (cdr l))))
```

. .

```
(define the-cons (cons 1 #f))
(set-cdr! the-cons the-cons)
```

```
(define (run-in-circles l)
  (+ (run-in-circles (cdr l))))
```

.."The program ran out of memory"

```
(define the-cons (cons 1 #f))
(set-cdr! the-cons the-cons)
```

(define (run-in-circles l)
 (run-in-circles (cdr l)))

```
(define the-cons (cons 1 #f))
(set-cdr! the-cons the-cons)
```

```
(define (run-in-circles l)
  (run-in-circles (cdr l)))
```

```
(define the-cons (cons 1 #f))
(set-cdr! the-cons the-cons)
```

```
(define (run-in-circles l)
  (run-in-circles (cdr l)))
```

```
(define the-cons (cons 1 #f))
(set-cdr! the-cons the-cons)
```

```
(define (run-in-circles l)
  (run-in-circles (cdr l)))
```

. .

```
(define the-cons (cons 1 #f))
(set-cdr! the-cons the-cons)
```

```
(define (run-in-circles l)
  (run-in-circles (cdr l)))
```

. . .

```
(define the-cons (cons 1 #f))
(set-cdr! the-cons the-cons)
```

```
(define (run-in-circles l)
  (run-in-circles (cdr l)))
```

```
(define the-cons (cons 1 #f))
(set-cdr! the-cons the-cons)
```

```
(define (run-in-circles l)
  (run-in-circles (cdr l)))
```

.

```
(define the-cons (cons 1 #f))
(set-cdr! the-cons the-cons)
```

```
(define (run-in-circles l)
  (run-in-circles (cdr l)))
```

.

```
(define the-cons (cons 1 #f))
(set-cdr! the-cons the-cons)
```

```
(define (run-in-circles l)
  (run-in-circles (cdr l)))
```

.

• What if we never had any deferred operations?

- What if we never had any deferred operations?
- Instead of *returning a value* with deferred operations, the function is passed a *continuation procedure*, which we call to return a value

- What if we never had any deferred operations?
- Instead of *returning a value* with deferred operations, the function is passed a *continuation procedure*, which we call to return a value
- Which means that all function calls are tail-recursive

```
(define (add-17 x)
(+ x 17))
```

```
(define (add-17 x)
  (+ x 17))
(define (add-17 x cont)
  (cont (+ x 17)))
```

Factorial in CPS

Factorial in CPS

Factorial in CPS

No deferred operations

- No deferred operations
- We craft a **new** continuation, based on the previous one, and pass that to our recursive call

- No deferred operations
- We craft a **new** continuation, based on the previous one, and pass that to our recursive call
- Asks the question, "What will I do with the return value of the recursive call?"

- No deferred operations
- We craft a **new** continuation, based on the previous one, and pass that to our recursive call
- Asks the question, "What will I do with the return value of the recursive call?"
- "Multiply it by n, and call my continuation with that value"

```
(define (sum-interval a b)
  (if (= a b))
      а
      (+ a (sum-interval (+ a 1) b))))
(define (cs-sum-interval a b cont)
  (if (= a b))
      (cont a)
      (cs-sum-interval
       (+ a 1)
        b
       (lambda (x) (cont (+ a x)))))
```

```
(define (append L1 L2)
 (if (null? L1)
      L2
      (cons (car L1) (append (cdr L1) L2))))
```

```
(define (append L1 L2)
  (if (null? L1)
     L2
      (cons (car L1) (append (cdr L1) L2))))
(define (cs-append L1 L2 cont)
  (if (null? L1)
      (cont L2)
      (cs-append
       (cdr L1)
       T.2
       (lambda (appended-cdr)
        (cons (car L1) appended-cdr)))))
```

```
(define (append L1 L2)
  (if (null? L1)
     L2
      (cons (car L1) (append (cdr L1) L2))))
(define (cs-append L1 L2 cont)
  (if (null? L1)
      (cont L2)
      (cs-append
       (cdr L1)
       T.2
       (lambda (appended-cdr)
        (cont (cons (car L1) appended-cdr))))))
```

```
(define (flatten tree)
  (cond ((null? tree) '())
        ((not (pair? tree)) (list tree))
        (else (append (flatten (car tree))
                       (flatten (cdr tree))))))
(define (cs-flatten tree cont)
  (cond ((null? tree) (cont '()))
        ((not (pair? tree)) (cont (list tree)))
        (else (cs-flatten
          (car tree)
          (lambda (car-leaves)
            (cs-flatten
             (cdr tree)
             (lambda (cdr-leaves)
                (cont.
                 (append car-leaves cdr-leaves)
 )))))))))
```

```
(define (flatten tree)
  (cond ((null? tree) '())
        ((not (pair? tree)) (list tree))
        (else (append (flatten (car tree))
                       (flatten (cdr tree))))))
(define (cs-flatten tree cont)
  (cond ((null? tree) (cont '()))
        ((not (pair? tree)) (cont (list tree)))
        (else (cs-flatten
          (car tree)
          (lambda (car-leaves)
            (cs-flatten
             (cdr tree)
             (lambda (cdr-leaves)
                (cont.
                 (append car-leaves cdr-leaves)
 )))))))))
```

```
(define (flatten tree)
  (cond ((null? tree) '())
        ((not (pair? tree)) (list tree))
        (else (append (flatten (car tree))
                       (flatten (cdr tree))))))
(define (cs-flatten tree cont)
  (cond ((null? tree) (cont '()))
        ((not (pair? tree)) (cont (list tree)))
        (else (cs-flatten
          (car tree)
          (lambda (car-leaves)
            (cs-flatten
             (cdr tree)
             (lambda (cdr-leaves)
                (cont.
                 (append car-leaves cdr-leaves)
 )))))))))
```
Flatten

```
(define (flatten tree)
  (cond ((null? tree) '())
        ((not (pair? tree)) (list tree))
        (else (append (flatten (car tree))
                       (flatten (cdr tree))))))
(define (cs-flatten tree cont)
  (cond ((null? tree) (cont '()))
        ((not (pair? tree)) (cont (list tree)))
        (else (cs-flatten
          (car tree)
          (lambda (car-leaves)
            (cs-flatten
             (cdr tree)
             (lambda (cdr-leaves)
                (cont.
                 (append car-leaves cdr-leaves)
 )))))))))
```

Flatten

```
(define (flatten tree)
  (cond ((null? tree) '())
        ((not (pair? tree)) (list tree))
        (else (append (flatten (car tree))
                       (flatten (cdr tree))))))
(define (cs-flatten tree cont)
  (cond ((null? tree) (cont '()))
        ((not (pair? tree)) (cont (list tree)))
        (else (cs-flatten
          (car tree)
          (lambda (car-leaves)
            (cs-flatten
             (cdr tree)
             (lambda (cdr-leaves)
                (cont.
                 (append car-leaves cdr-leaves)
 )))))))))
```

Flatten

```
(define (flatten tree)
  (cond ((null? tree) '())
        ((not (pair? tree)) (list tree))
        (else (append (flatten (car tree))
                       (flatten (cdr tree))))))
(define (cs-flatten tree cont)
  (cond ((null? tree) (cont '()))
        ((not (pair? tree)) (cont (list tree)))
        (else (cs-flatten
          (car tree)
          (lambda (car-leaves)
            (cs-flatten
             (cdr tree)
             (lambda (cdr-leaves)
                (cont.
                 (append car-leaves cdr-leaves)
 )))))))))
```

 Continuation-passing style is also very useful in controlling program flow

- Continuation-passing style is also very useful in controlling program flow
- Error handling and exceptions is a classic case:

- Continuation-passing style is also very useful in controlling program flow
- Error handling and exceptions is a classic case:

```
(define (divide a b success fail)
 (if (= b 0)
    (fail "divide-by-zero")
    (success (/ a b))))
```

- Continuation-passing style is also very useful in controlling program flow
- Error handling and exceptions is a classic case:

```
(define (divide a b success fail)
 (if (= b 0)
      (fail "divide-by-zero")
      (success (/ a b))))
```

Also asynchronous procedure calls

Continuations in the interpreter

We can write a Scheme interpreter in continuation-passing style

We can write a Scheme interpreter in continuation-passing style

```
(define (driver-loop)
  (prompt-for-input input-prompt)
  (let ((input (read)))
    (if (eq? input '**quit**)
        'c-eval-done
        (c-eval
         input
         the-global-environment
         (lambda (output)
           (announce-output output-prompt)
           (display output)
           (driver-loop))))))
```

```
(define (c-eval exp env cont)
 (cond ((self-evaluating? exp)
         (cont exp))
        ((variable? exp)
         (cont (lookup-variable-value exp env)))
        ((quoted? exp)
         (cont (text-of-quotation exp)))
        ((assignment? exp)
         (eval-assignment exp env cont))
        ((definition? exp)
         (eval-definition exp env cont))
        ((if? exp) (eval-if exp env cont))
        ((lambda? exp)
         (cont (make-procedure (lambda-parameters exp)
                                (lambda-body exp) env)))
        ((begin? exp)
         (eval-sequence (begin-actions exp) env cont))
        ((cond? exp)
         (c-eval (cond->if exp) env cont))
```

```
(define (c-eval exp env cont)
 (cond ((self-evaluating? exp)
         (cont exp))
        ((variable? exp)
         (cont (lookup-variable-value exp env)))
        ((quoted? exp)
         (cont (text-of-quotation exp)))
        ((assignment? exp)
         (eval-assignment exp env cont))
        ((definition? exp)
         (eval-definition exp env cont))
        ((if? exp) (eval-if exp env cont))
        ((lambda? exp)
         (cont (make-procedure (lambda-parameters exp)
                                (lambda-body exp) env)))
        ((begin? exp)
         (eval-sequence (begin-actions exp) env cont))
        ((cond? exp)
         (c-eval (cond->if exp) env cont))
```

```
(define (c-eval exp env cont)
 (cond ((self-evaluating? exp)
         (cont exp))
        ((variable? exp)
         (cont (lookup-variable-value exp env)))
        ((quoted? exp)
         (cont (text-of-quotation exp)))
        ((assignment? exp)
         (eval-assignment exp env cont))
        ((definition? exp)
         (eval-definition exp env cont))
        ((if? exp) (eval-if exp env cont))
        ((lambda? exp)
         (cont (make-procedure (lambda-parameters exp)
                                (lambda-body exp) env)))
        ((begin? exp)
         (eval-sequence (begin-actions exp) env cont))
        ((cond? exp)
         (c-eval (cond->if exp) env cont))
```

```
(define (c-eval exp env cont)
 (cond ((self-evaluating? exp)
         (cont exp))
        ((variable? exp)
         (cont (lookup-variable-value exp env)))
        ((quoted? exp)
         (cont (text-of-quotation exp)))
        ((assignment? exp)
         (eval-assignment exp env cont))
        ((definition? exp)
         (eval-definition exp env cont))
        ((if? exp) (eval-if exp env cont))
        ((lambda? exp)
         (cont (make-procedure (lambda-parameters exp)
                                (lambda-body exp) env)))
        ((begin? exp)
         (eval-sequence (begin-actions exp) env cont))
        ((cond? exp)
         (c-eval (cond->if exp) env cont))
```

```
(define (eval-if exp env cont)
 (c-eval
  (if-predicate exp) env
  (lambda (test-value)
      (if test-value
            (c-eval (if-consequent exp) env cont)
                  (c-eval (if-alternative exp) env cont)))))
```

```
(define (eval-if exp env cont)
  (c-eval
   (if-predicate exp) env
   (lambda (test-value)
     (if test-value
         (c-eval (if-consequent exp) env cont)
         (c-eval (if-alternative exp) env cont)))))
(define (eval-sequence exps env cont)
  (if (last-exp? exps)
      (c-eval (first-exp exps) env cont)
      (c-eval (first-exp exps) env
       (lambda (ignored)
         (eval-sequence
          (rest-exps exps)
          env cont)))))
```

Continuations with the interpreter

• What if the evaluator made its continuations available to the language?

Continuations with the interpreter

- What if the evaluator made its continuations available to the language?
- call-with-current-continuation (a.k.a. call/cc)

Continuations with the interpreter

- What if the evaluator made its continuations available to the language?
- call-with-current-continuation (a.k.a. call/cc)

```
;; in c-apply
```

```
(+ (* 3 (call-with-current-continuation
                      (lambda (cont)
                      (cont 5))))
10)
; => 25
(define c #f)
(+ (* 3 (call-with-current-continuation
                     (lambda (cont)
                     (set! c cont)
                     (cont 5))))
10)
```

```
(+ (* 3 (call-with-current-continuation
          (lambda (cont)
            (cont 5))))
   10)
; => 25
(define c #f)
(+ (* 3 (call-with-current-continuation
          (lambda (cont)
            (set! c cont)
            (cont 5))))
   10)
; => 25
```

```
(+ (* 3 (call-with-current-continuation
          (lambda (cont)
            (cont 5))))
   10)
; => 25
(define c #f)
(+ (* 3 (call-with-current-continuation
          (lambda (cont)
            (set! c cont)
            (cont 5))))
   10)
; => 25
(c 6)
```

```
(+ (* 3 (call-with-current-continuation
          (lambda (cont)
            (cont 5))))
   10)
; => 25
(define c #f)
(+ (* 3 (call-with-current-continuation
          (lambda (cont)
            (set! c cont)
            (cont 5))))
   10)
; => 25
(c 6)
; => 28
```

```
(+ (* 3 (call-with-current-continuation
         (lambda (cont)
            (cont 5))))
   10)
; => 25
(define c #f)
(+ (* 3 (call-with-current-continuation
         (lambda (cont)
            (set! c cont)
            (cont 5))))
  10)
; => 25
(c 6)
; => 28
(+100 (c 6))
```

```
(+ (* 3 (call-with-current-continuation
         (lambda (cont)
            (cont 5))))
   10)
; => 25
(define c #f)
(+ (* 3 (call-with-current-continuation
         (lambda (cont)
            (set! c cont)
            (cont 5))))
  10)
; => 25
(c 6)
; => 28
(+ 100 (c 6))
; => 28
```

• call-with-current-continuation (or call/cc, as it is usefully shortened to) takes a procedure as an argument, and passes it the evaluator's current continuation

- call-with-current-continuation (or call/cc, as it is usefully shortened to) takes a procedure as an argument, and passes it the evaluator's current continuation
- The return value of call/cc is the same as the return value of the procedure

- call-with-current-continuation (or call/cc, as it is usefully shortened to) takes a procedure as an argument, and passes it the evaluator's current continuation
- The return value of call/cc is the same as the return value of the procedure
- ... or the procedure could just call the continuation it was given. Which is exactly identical in meaning!

- call-with-current-continuation (or call/cc, as it is usefully shortened to) takes a procedure as an argument, and passes it the evaluator's current continuation
- The return value of call/cc is the same as the return value of the procedure
- ... or the procedure could just call the continuation it was given. Which is exactly identical in meaning!
- The continuation of the call/cc expression, the continuation of the procedure that it calls, and the **value** that it passes as an argument to that procedure, are all the same!

```
(define (fib-func)
 (let ((prev 0)
        (cur 1))
   (define (loop)
        (define next (+ prev cur))
        (set! prev cur)
        (set! cur next)
        prev)
        loop))
```
```
(define (fib-func)
 (let ((prev 0)
        (cur 1))
    (define (loop)
        (define next (+ prev cur))
        (set! prev cur)
        (set! cur next)
        prev)
        loop))
(define test (fib-func))
```

```
(define (fib-func)
  (let ((prev 0)
        (cur 1))
    (define (loop)
        (define next (+ prev cur))
        (set! prev cur)
        (set! cur next)
        prev)
        loop))
(define test (fib-func))
(test) ; => 1
```

```
(define (fib-func)
  (let ((prev 0)
        (cur 1))
    (define (loop)
      (define next (+ prev cur))
      (set! prev cur)
      (set! cur next)
     prev)
    loop))
(define test (fib-func))
(test) ; => 1
(test); => 1
```

```
(define (fib-func)
  (let ((prev 0)
        (cur 1))
    (define (loop)
      (define next (+ prev cur))
      (set! prev cur)
      (set! cur next)
     prev)
    loop))
(define test (fib-func))
(test) ; => 1
(test) ; => 1
(test); => 2
```

```
(define (fib-func)
  (let ((prev 0)
        (cur 1))
    (define (loop)
      (define next (+ prev cur))
      (set! prev cur)
      (set! cur next)
     prev)
    loop))
(define test (fib-func))
(test); => 1
(test) ; => 1
(test); => 2
(test) ; => 3
```

```
(define (fib-func)
  (let ((prev 0)
        (cur 1))
    (define (loop)
      (define next (+ prev cur))
      (set! prev cur)
      (set! cur next)
     prev)
    loop))
(define test (fib-func))
(test); => 1
(test) ; => 1
(test) ; => 2
(test) ; => 3
(test) ; => 5
```

```
(define resume 'uninitialized)
(define (fib-cont)
  (let ((prev 0)
        (cur 1))
    (define (loop)
      (define next (+ prev cur))
      (set! prev cur)
      (set! cur next)
      (if (call/cc
           (lambda (c)
              (set! resume (lambda () (c #f)))
             (c #t)))
          prev
          (loop)))
    (loop)))
```

```
(define resume 'uninitialized)
(define (fib-cont)
  (let ((prev 0)
        (cur 1))
    (define (loop)
      (define next (+ prev cur))
      (set! prev cur)
      (set! cur next)
      (if (call/cc
           (lambda (c)
              (set! resume (lambda () (c #f)))
             (c #t)))
          prev
          (loop)))
    (loop)))
(fib-cont); => 1
```

```
(define resume 'uninitialized)
(define (fib-cont)
  (let ((prev 0)
       (cur 1))
    (define (loop)
      (define next (+ prev cur))
      (set! prev cur)
      (set! cur next)
      (if (call/cc
           (lambda (c)
             (set! resume (lambda () (c #f)))
             (c #t)))
          prev
          (loop)))
    (loop)))
(fib-cont); => 1
(resume) ; => 1
```

```
(define resume 'uninitialized)
(define (fib-cont)
 (let ((prev 0)
       (cur 1))
    (define (loop)
      (define next (+ prev cur))
      (set! prev cur)
      (set! cur next)
      (if (call/cc
           (lambda (c)
             (set! resume (lambda () (c #f)))
             (c #t)))
          prev
          (loop)))
    (loop)))
(fib-cont); => 1
(resume) ; => 1
(resume) ; => 2
```

```
(define resume 'uninitialized)
(define (fib-cont)
 (let ((prev 0)
       (cur 1))
    (define (loop)
      (define next (+ prev cur))
      (set! prev cur)
      (set! cur next)
      (if (call/cc
           (lambda (c)
             (set! resume (lambda () (c #f)))
             (c #t)))
          prev
          (loop)))
    (loop)))
(fib-cont); => 1
(resume) ; => 1
(resume) ; => 2
(resume) ; => 3
```

```
(define resume 'uninitialized)
(define (fib-cont)
  (let ((prev 0)
        (cur 1))
    (define (loop)
      (define next (+ prev cur))
      (set! prev cur)
      (set! cur next)
      (if (call/cc
           (lambda (c)
             (set! resume (lambda () (c #f)))
             (c #t)))
          prev
          (loop)))
    (loop)))
(fib-cont); => 1
(resume) ; => 1
(resume) ; => 2
(resume) ; => 3
(resume) ; => 5
```

• Save the continuation, return true now

- Save the continuation, return true now
- But call the continuation with **false** again, sometime in the future, to take the other branch

- Save the continuation, return true now
- But call the continuation with **false** again, sometime in the future, to take the other branch
- In this case, resumes the loop!

- Save the continuation, return true now
- But call the continuation with **false** again, sometime in the future, to take the other branch
- In this case, resumes the loop!
- This pattern is known as a coroutine

- Save the continuation, return true now
- But call the continuation with **false** again, sometime in the future, to take the other branch
- In this case, resumes the loop!
- This pattern is known as a coroutine
- Poor man's threading (running multiple things at once)

- Save the continuation, return true now
- But call the continuation with **false** again, sometime in the future, to take the other branch
- In this case, resumes the loop!
- This pattern is known as a coroutine
- Poor man's threading (running multiple things at once)
- ... but we can do better...

 Only one bit of code can run at once, but we have multiple tasks to do

- Only one bit of code can run at once, but we have multiple tasks to do
- Make each task declare when it's done doing some computation, and then swap

- Only one bit of code can run at once, but we have multiple tasks to do
- Make each task declare when it's done doing some computation, and then swap
- "Co-operative" because tasks need to declare when they want to let someone else have a turn

- Only one bit of code can run at once, but we have multiple tasks to do
- Make each task declare when it's done doing some computation, and then swap
- "Co-operative" because tasks need to declare when they want to let someone else have a turn
- Used by Mac OS 9, Windows 3.1